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SO, YOU WANT TO BE AN INVENTOR

BY

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It is an appropriate time to talk about your aspirations to be an inventor because we have just observed the 500th anniversary of the birth of one of the greatest, Leonardo da Vinci. Although best known for great paintings and superb drawings of the human anatomy, Leonardo might have achieved lasting fame for his scores of brilliant inventions. Maybe you think that air-conditioning is an innovation of our times? Leonardo, history reports, built an air-conditioning unit for the boudoir of Beatrice d'Este, the wife of his patron, about the end of the 15th Century. Many people attribute the machine gun to Gattling or to Sir Hiram Maxim, but Leonardo had a couple of not-too-bad machine gun designs back in his day.

I recall going across a two level bridge at Spuyten Duyvil, New York, a few times last summer and feeling thankful that the traffic was in one direction only. This summer when I leave Manhattan Island via the two level bridge I'll remember that Leonardo proposed a two-level bridge—upper level for pedestrians and lower level for vehicles. Perhaps you could make a great invention by borrowing from Leonardo and devising a simple, cheap, and practical way to redesign a few of our city streets with a pedestrian level and a motorcar level. Two groups of our population would be eternally grateful, even if the idea lacks novelty.

If you want to be an inventor, you might use a few broad hints. Without even the slightest pretense of modesty, I think I am your man. My daily job has put me in a position of seeing not only hundreds of the brain children of some of the world's best electronic scientists and engineers, but thousands of the products of the minds of many enthusiastic "outsiders" who "want to be inventors." Every mail brings more ideas but rarely an invention, much less a patentable one. Why do so

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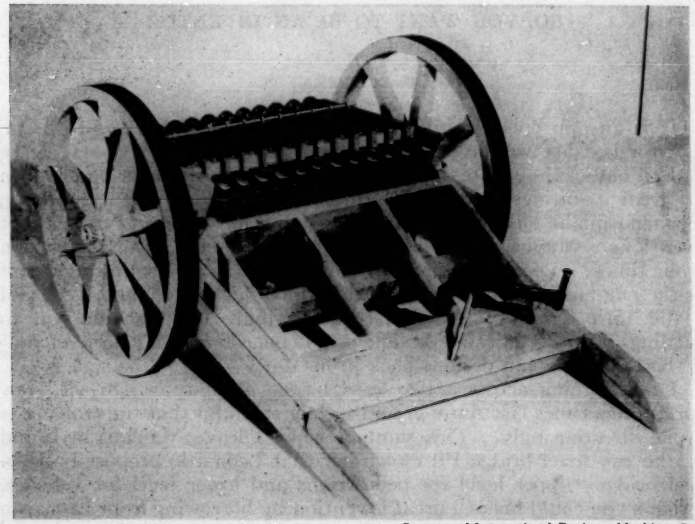
many people have an apparently irresistible urge to create inventions? What are some of the conditions that beget invention? I am going to take a careful look at the circumstances surrounding the birth of a few of the inventive creations of inventors well known to many of us. In doing so I may be fortunate enough to discover a few "rules for inventors" but at least I shall make observations that may be helpful.

If this were a dignified logical approach, you would expect me to define what I mean by inventor, invention, discovery, patentable invention, and perhaps a few other terms of the patent profession. The



Courtesy of International Business Machines Corp.

Photograph of a model of Leonardo da Vinci's triple-tier machine gun, built by Roberto A. Guatelli from original drawings. The instrument had twelve guns in each of its three tiers. One tier was to be fired while a second was being loaded and the third was cooling.

astounding thing is that if I tried to define the terms, I would be rushing in where many of the great ones hesitate to tread. Even the most able of our judges have concluded that invention slyly eludes definition. I am content, for the present, to let it go at that because it won't help to say an inventor is one who invents. But it may help to look back at a few recognized inventors and how some of them came to invent or discover some of the things that made them famous.

Thinking that age might be a factor in invention I picked about a dozen and a half notable inventions that arrived during the 150 years

following James Watts' steam engine. An even dozen of the inventors were between 27 and 35 years old when they made their important contribution. If you happen to be younger or older don't be impatient or discouraged. Marconi made one of his great wireless contributions at age 21 according to my calculations; McCormick came along with a successful grain reaper at 22; Sholes was 48 before he contributed his part of the typewriter; and Sir Hiram Maxim produced his machine gun when he was almost 50. I have no doubt Edison was not to be treated lightly as an inventor in his ripe old age—patents continued to issue in his name after his death at 84. Even teen agers have come up with things I thought involved invention. I conclude that inventions and discoveries come to young and old—so don't worry too much about your age if you are receptive to invention.

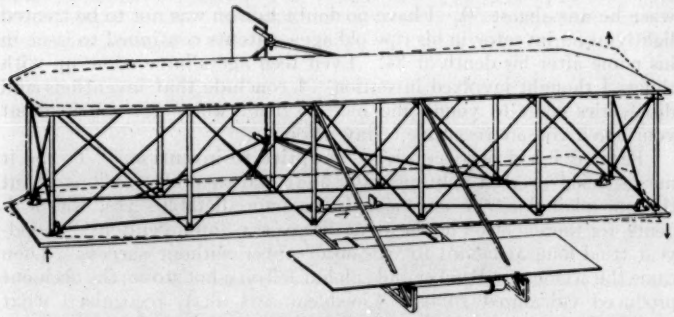
Perhaps I had better explain "receptive to invention." To me it means being everlastingly mentally alert and mentally curious about the unexplainable, the unexpected, the unpredictable—yes, the accidents, for therein often lie the great discoveries and inventions. Goodyear tried long and hard to vulcanize rubber without success. Then came the accident. Rubber and sulphur fell on a hot stove; the accident produced vulcanized rubber. Goodyear was alert, recognized what happened, and made a "discovery" ("discovery" and "invention" are treated synonymously in our patent law).

One of Edison's greats, the phonograph, came from his insatiable search for an explanation. He was working on his telegraph repeater that included a motor-driven cylinder. The motor had gone berserk and a contact on the cylinder produced strange and most undesirable sounds. While his immediate purpose was to fix the machine, he was unable to put aside the thought that the combination might have other uses. Perhaps he could modify the surface of the cylinder to produce a desired sound? He experimented with surface materials; found tinfoil suitable; and on August 22, 1877 requested John Kruesi to make a manually rotatable cylinder and a stylus arrangement. Edison applied tinfoil to the cylinder; shouted through the mouthpiece, thus actuating the stylus, and to everyone's surprise the machine talked back. Thus, the phonograph was born when Thomas Edison was 30 years old. Incidentally, patentwise he landed in a virgin field. Do I hear shouts of anguish from beyond the Iron Curtain?

Most inventions selected for this review did not arise by accident or in a virgin field. I do not recommend that you just sit waiting for accidents to happen—but do not overlook the latent possibilities of the unexpected. Perhaps we have arrived at the educational stage when deliberate, thoughtful approach to invention is most apt to pay off. I have in mind Wilbur and Orville Wright. When they made their first successful flight and landing at Kitty Hawk, around the beginning of our century, it was not on a virgin field of invention.

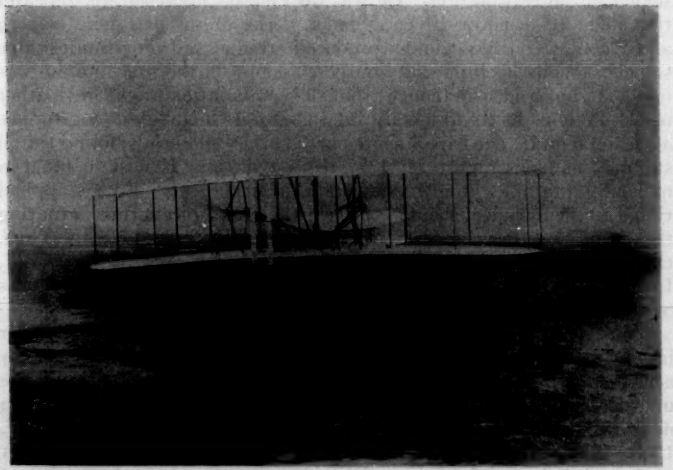
The Wrights had been aware of the many successful glider flights of Otto Lilienthal. They knew that he shifted his weight to balance his glider. They were satisfied that the shifting weight idea provided insufficient control and was probably the cause of the gliding accident

FIG. 1.



Mankind tried to imitate the flight of the birds from long prior to Leonardo da Vinci, but it remained for the Wright Brothers to master the mystery of aerodynamic control. This is a reproduction of Fig. 1 of O. and W. Wright, U. S. Patent No. 821,393, issued May 22, 1906.

V. 19

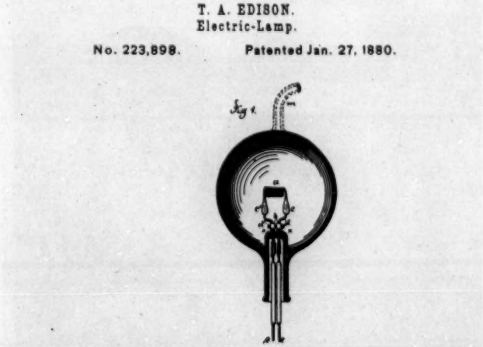


Courtesy of the Smithsonian Institution

Photograph of the first Wright Brothers' plane.

resulting in Lilienthal's death. Wrights found existing scientific data on airfoils unreliable. The two brothers devised and operated a small wind tunnel. Their wind tunnel experiments determined the lift and drag of many differently shaped airfoils operated at different angles. They developed a system of wing warping and of varying air pressures to provide the necessary equilibrium.

In October 1902 they designed a heavier-than-air flying machine. The machine was powered by a 12-horsepower, 4-cylinder gasoline motor. The airplane, including pilot, weighed 750 pounds. Orville successfully flew the machine at Kitty Hawk, N. C., on December 17, 1903. The flight lasted 59 seconds at a speed of 30 miles per hour. By October 5, 1905, Wilbur flew for 38 minutes over a small circular course covering 24 miles. Thus we have one example of a painstaking



Edison's invention of the electric-lamp had a greater effect on civilization during his lifetime than all illuminating inventions from the beginning of time to Edison's birth.

investigation that resulted in a successful invention where Leonardo, Lilienthal and many others tried with doubtful success or failed entirely.

There is another approach to invention that Edison used on occasion. We might call it a "brute force" approach. The majority of electrical engineers thought that electric lights must be operated by sending current through the illuminating devices in series. This required low resistance filaments that dissipated a heavy current. Edison courageously decided that the majority were in error. He started a search for a suitable filament. The filament had to be of small diameter, high resistance, and of long life. Some carbonized threads showed promise. He sent men all over the world looking for fibres that might have the necessary characteristics when carbonized. By sheer numbers of experiments and effort he succeeded about 1879. It now looks like a

brute force approach and, perhaps there was no other. Also do not forget that he went contrary to the pack.

On occasion the invention *seems* to come so easily—once the need is clearly recognized *and the solution vigorously sought.* McCormick's Grain Reaper is an example. Cyrus Hall McCormick was the son of a farmer who had invented numerous devices for farm use. The father made repeated, but unsuccessful efforts, to build a grain-cutting device. In 1831, at the age of 22, young Cyrus took up the problem. He developed the reaper which was used successfully in the harvest of 1831 and was patented in 1834.

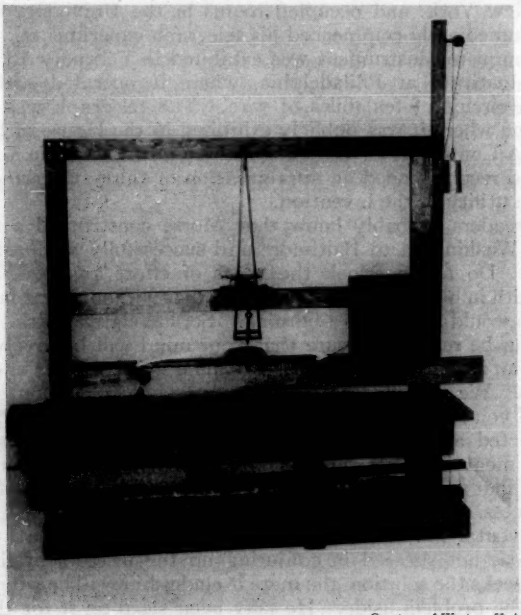
Courtesy of International Harvester Co.

With Cyrus McCormick's reaper, two people, a man or boy who rode the horse and a man who raked the cut grain off the platform, replaced four or five men cutting grain with cradles or twelve to sixteen men with hand sickles.

When Eli Whitney graduated from Yale in 1792, he had already demonstrated mechanical ability. Mrs. Nathaniel Greene, widow of the Revolutionary War General, invited Whitney to Savannah. While there and considering his future course, he made several ingenious household contrivances. He was introduced to some gentlemen who were discussing the need for a machine to separate cotton from its seeds. Within a few weeks Whitney produced a model. It consisted of a wooden cylinder carrying rows of spokes that extended through a comb set to pull the lint but not passing the seed. The model was operated by hand and was capable of separating 50 pounds of cotton lint per day.

Thus the reaper and the cotton gin inventions appear to have been born painlessly.

Sometimes the mental act of inventing unfolds itself quite quickly and completely. Nevertheless, the actual operable device is developed very slowly. Lack of funds, facilities, or many other difficulties seem to intervene. According to Supreme Court records Samuel Morse, by profession a historical painter, was returning from Europe in 1832 on



Courtesy of Western Union Telegraph Co

First crude telegraph constructed by S. F. B. Morse at New York University (Washington Square), in 1835.

the packet ship *Sully*. The brilliant conversations of his fellow shipmates as to electrical matters helped him to see that by means of electricity, signs representing figures, letters, or words, might be reproduced at any distance.

Before he landed in the United States, he had conceived and drawn out in his sketch book, the form of an instrument for an electromagnetic telegraph. Morse had arranged and noted down a system of signs composed of a combination of dots and spaces, which were to represent

figures. These were to indicate words to be found in a telegraphic dictionary, where each word was to have its number.

Unable to purchase the materials for a battery and wire, Morse was compelled, for subsistence, to return to his art. From 1832 to the latter part of 1835, there was no opportunity to test the invention. He never lost faith in its practicability, nor abandoned his intention of testing it as soon as he could command the means.

In 1835 Morse was appointed Professor in the University of the City of New York, and occupied rooms in the University buildings. Here he immediately commenced his telegraph experiments.

The completed instrument was exhibited in February 1838 in The Franklin Institute at Philadelphia, where it operated with success through a circuit of ten miles of wire. The telegraph was taken to Washington where it was publicly exhibited in the House of Representatives. A Committee of the Congress examined it, made a favorable report, and recommended an appropriation of thirty thousand dollars, to test the utility of the invention.

Most readers probably know that Morse constructed a telegraph line from Washington to Baltimore and successfully demonstrated its operation. Do not overlook the years of effort and the inventor's implicit faith in his invention. I am sure that without great faith many inventions would have died a-borning. So, have great faith.

You can be reasonably sure that your mind will be involved when you make an invention because it is the mind which creates or conceives or devises. We do not need to get too psychological about it, but it will help if you realize that the process may be deliberate and conscious, or unexpected and subconscious. Although inventors rarely tell us about the mental process through which their inventions were coined, I suspect that not infrequently inventions arrive through a subconscious process.

The inventor may start by recognizing a clear need but, try as he will, he does not succeed in conjuring up the instrumentality. The harder he seeks the solution, the more it eludes him. The wise inventor may put the problem aside. He may even sleep on it for that often seems to help. But, whatever the process, he thinks of something else and the first thing he knows the answer arrives by some little understood subconscious mechanism.

I recall Hiram Percy Maxim, son of Sir Hiram and nephew of Hudson Maxim, telling about his invention of the gun silencer. He liked to shoot at a target with a small rifle but he did not like to annoy his neighbors. If he could only put something on the rifle to eliminate the noise, target shooting would be so much more enjoyable. He did not have the vaguest idea of how to accomplish the desired result.

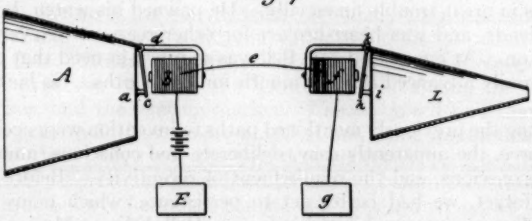
One morning after shaving, Maxim released the water from the lavatory. The water did not suddenly disappear but after a rotary

action, went slowly down the drain. He quickly realized that if the gases from the exploding cartridge in the rifle could be made to expand slowly by a rotary motion, instead of suddenly being released from the muzzle, the noise would be reduced. It did not take long to make a cylindrical device with a straight hole for the rifle bullet and a tortuous expansion space for rotating the gas and trapping the sound. Thus the gases came out slowly and the noise was silenced. Maxim may have arrived at the result by the subconscious process. Some reasoning by analogy was also involved.

Seeking advice sometimes encourages an inventor. If you decide on that course, go to the truly great for they can afford to render sound, honest judgment. Alexander Graham Bell might not have gone down in history as the inventor of the telephone had he not consulted the famous Professor Joseph Henry.

Bell, conceiving the speaking telephone in the year 1874, had serious doubts that the device would be practical because of the feebleness of

Fig. 7



Bell's Singing Telegraph Patent (No. 174,465, dated March 7, 1876) was the forerunner of his telephone patent of 1877. An accidental vibration of the spring *c* gave Bell the clue for the invention of the telephone. This is a reproduction of Fig. 7 of the Telegraph patent.

the voice-generated electric currents. After explaining his views to Professor Henry in 1875, Bell wrote to his father and mother as follows:

I felt so much encouraged by his interest, that I determined to ask his advice about the apparatus I have designed for the transmission of the human voice by telegraph. I explained the idea, and said, "What would you advise me to do; publish it and let others work it out, or attempt to solve the problem myself?" He said he thought it was the germ of a great invention, and advised me to work at it myself, instead of publishing. I said that I recognized the fact that there were mechanical difficulties in the way that rendered the plan impracticable at the present time. I added that I felt that I had not the electrical knowledge necessary to overcome the difficulties. His laconic answer was, "GET IT."

I cannot tell you how much those two words have encouraged me. I live too much in an atmosphere of discouragement for scientific pursuits. Good (Mr.)_____ is unfortunately one of the *cui bono* people, and is too much in the habit of looking at the dark side of things. Such a chimerical idea as telegraphing *vocal sounds* would indeed, to *most minds*, seem scarcely feasible enough to spend the time in working over. I believe, however, that it is feasible, and that I have got the clue to the solution of the problem.

Note also the "accidental" approach in Bell's work on the telephone. His attorney arguing before the U. S. Supreme Court said this about Bell:

> In performing . . . an experiment with a . . . telegraph which employed two reeds or springs vibrated in front of an electromagnet, one of the springs was accidentally knocked, and thus set in vibration. He found that this slight vibration produced a sound from the spring of another instrument connected in electrical circuit. With another man the trivial accident might have passed unnoticed. But he instantly joined it with his older thoughts. The marriage was fruitful and the speaking telephone was born. It thenceforth needed only nurture. It at once struck him that if he was right in his observation of this accident, then the feeble vibrations of a spring in front of an electro-magnet had developed sufficient electric currents to produce audible sonorous effects at a distance. He repeated the experiment . . . , sanguinely satisfied that his former fears about the feebleness of the currents were ill founded, he instantly gave orders for the construction of a speaking telephone with a membrane diaphragm. . . .

I do not wish to overlook the courage frequently found in inventors. Bell was an example. The first instruments, illmade, broke on the first trial. They were repaired without much success. It is not certain that a single spoken word was understood during the early experiments. Bell was in great trouble financially. He pawned his watch, borrowed from friends, and was heart-broken for other reasons. Nevertheless, he kept on. At one period Mr. Bell was so much in need that a friend of the family advanced $25 per month for six months. So lack not in courage.

Among the previously mentioned paths to invention were accidental, brute force, the apparently easy, deliberate and conscious, unexpected and subconscious, and the requirement of receptivity. Before I completely forget, we had better get to persistence, which many of our outstanding inventors exhibited in force. Bell, Edison, Morse, and the Wright Brothers were certainly persistent, and you know where it got them.

John Philip Holland's invention of the submarine wins my nomination for unabated persistence. The Civil War engagement of the *Merrimac* and the *Monitor* suggested the submarine to Holland. His first underwater boat was built in 1875. It failed, as did Holland's second, third, fourth, fifth, sixth, seventh, and eighth boats. His ninth submarine, finished in 1898 after 23 years of untiring efforts, turned out to be serviceable! So, be persistent.

The reader may wonder if invention demands a particular education. I doubt that a study of the 19th and early 20th century inventors' education will be very helpful in forecasting the requirements of the last half of the 20th century. We know that Thomas Edison, Hudson Maxim, James Watt, Sir Hiram Maxim, Elias Howe, and the Wright Brothers, just to mention a few outstanding inventors, lacked a "formal" education. Guglielmo Marconi and Alfred Nobel, the inventor of dynamite, were privately educated. On the other hand Eli Whitney,

Lee de Forest, Samuel Morse, Hiram Percy Maxim, and others equally notable were college graduates. It was probably a matter of intelligence rather than where or how the education was obtained.

From what I've read of Henry Ford, I'd conclude that he thought formal education might be dangerous to originality and of that we know he possessed plenty. However, I'd really be surprised if his grandson Henry Ford II decided that men with a college degree were incapable of refreshing originality. I believe we are entering an age when true scientific knowledge will be a tremendous factor in invention and discovery. But I do not believe that you are ruled out if you got your knowledge "the hard way"—provided you got it! Remember Professor Henry's advice to Bell: "GET IT!"

I can just hear some impatient reader snort: "Enough of this psychological and philosophical verbage! Tell us what to invent, or how to invent." Well it may be that I have used different words, but I have tried to point out some of the ambient circumstances that may help you to be receptive to invention, and that is as near as I propose to come to any specific discourse on "how to invent."

At least one book has been written with many suggestions of "what to invent" and I suppose a diligent search would disclose other "please invent" lists. Before you undertake casually to fill these expressed needs, I urge you to study the literature, the patents, the previous approaches, and the waiting market. Usually you'll find that others have proposed solutions—some practical and some impractical. You may ask "Why have not the practical solutions been adopted and marketed?" Some may have been too early—before their time. The inventors may have found the commercial path too trying, or too difficult, or too long.

Eli Whitney tried to market the cotton gin but he had infringers galore. Whitney became involved in patent litigation. Although his patent was finally sustained, Whitney had given up in disgust and started to manufacture firearms for the United States Government. Even at that Whitney could not shut off his capacity for originality for he is credited with introducing factory division of labor and standardization of parts.

What to invent? Fortunately, the area of invention is as boundless as the universe. I do not suppose that there is an industry, an art, a science, a business, or any sphere of human endeavor that is not at this very moment crying for originality, for new things, for new methods. A catalogue of even a few of the inventions and discoveries as yet unborn would fill many pages of fine print.

General David Sarnoff, at a luncheon in honor of his forty-five years in radio, asked for three inventions as presents for his fiftieth anniversary. You may be interested because as a bystander you might have thought that each of the inventions was already in use. In a manner

of speaking they are, but what General Sarnoff requested will demand no small measure of perseverance and a great measure of originality.

I doubt if he is much concerned with the order of their appearance. So here goes: First, a practical and truly novel method of recording the video portion of television programs so that they can be reproduced with fidelity and at low cost anywhere at any time. Second, a true amplifier of light that will make possible inexpensive projection of television pictures in the home. There must be adequate light, rapidity of control, and distortionless enlargement. Third, an electronic air-conditioner that will provide for the home natural air conditioning with a minimum of noise and preferably without any mechanically moving parts. I may have included a few limitations that General Sarnoff tacitly implied. Incidentally, he only allowed five years, so you had better get busy! In the meantime industry waits.

I do not have any anniversary coming up, I have earned no right to request presents, and besides my wants are so very simple. I'll just go to the nearest drug store and try to buy the unattainable—a practical, neat, sanitary, refillable, tooth paste dispenser with a cap that refuses to get lost. And please note that "refillable" is *not* synonymous with "collapsible" in my dictionary. By way of encouragement for the inventor—I shall not insist on an unhappy degree of cheapness! Will one of my readers just come up with that necessary and long awaited invention? So, you still want to be an inventor?

THE INFRARED TRANSMISSION OF ATMOSPHERIC WINDOWS*

BY

TAIT ELDER[1,2] AND JOHN STRONG[1]

ABSTRACT

This paper describes an approach to the practical problem of infrared atmospheric transmission. It compares and relates published data from diverse investigations.

The method separates water vapor band absorption, on which there appears to be general agreement, from the less tractable continuous attenuation by fog, haze, and smoke. This separation is effected by a smooth envelope curve drawn tangent to the principal maxima in the observed transmitted spectra. In each window, the monochromatic transmission at the window peak is shown to be representative of the effect of continuous absorption on the total transmission in the window region.

A simple empirical relation is developed to represent the dependence of the band absorption on the water path for each window.

The dependence of continuous attenuation upon the water path for clear atmosphere is described, and the effects of haze, fog, and smoke are very briefly discussed.

INTRODUCTION

A knowledge of the infrared transmission of the atmosphere is important in meteorology and for certain military problems, entirely apart from the purely physical interest which has prompted much investigation during the past 70 years.

The absorption of infrared radiation by the molecules of atmospheric gases has been more thoroughly studied than the attenuation by the particles of fog, haze, or smoke. The main effect of the dispersed liquid or solid atmospheric constituent particles is scattering. The theory of scattering by particles of known size and shape has been verified extensively by laboratory experiments, in both the visible and infrared regions. However, the strong dependence upon particle size and density precludes a simple application to the atmosphere, where mean particle sizes and size distributions vary greatly from place to place and with time. Furthermore, observations of attenuation due to particles, and necessarily also molecules, are often difficult to interpret in such a manner that the effects of particles as seen by various observers can be related. This is because published observations are usually characterized by measurements of either transmission or particle size, but seldom both.

In contrast, existing experimental studies of molecular absorption by water vapor, carbon dioxide, and ozone—the three principal absorb-

* This work was supported in part by ONR Contract Nonr 248(01).

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[2] Now at the New Jersey Zinc Company (of Pennsylvania), Palmerton, Pa.

ing atmospheric constituents—provide both the transmission spectra and the specified definite physical conditions of the pressure, temperature, and (usually) concentration of absorber gases which were involved. These studies, with few exceptions, provide significant physical data to be accounted for. It is only because of differences of presentation and notation between authors that intercomparisons have heretofore not been complete nor of a character to give confidence in extrapolation to physical conditions beyond those encompassed in the original work.

The fact that both the particle and molecular absorption problems are still being pursued attests to the inadequacy of the existing knowledge—especially the inadequacy for predicting the features of the infrared transmission in the atmosphere which are of current interest in meteorology.

On the instrumental side, the complexity of the subject so far transcends the resolving power of instruments that each new instrumental development vastly expands the work to be done (before the last word can be said). The great advances available in current instrumentation over that available to some of the pioneers, like Fowle, has discouraged intercomparisons.

Current theoretical developments can often treat special cases with some degree of success. The absorption due to one line, to a group of equally-spaced, equally-intense lines, or to a whole band, can, in principle, be represented theoretically. But because of the relatively complex spectra of the atmospheric absorbers, H_2O and CO_2, and the uncontrolled and variable physical parameters (such as pressure), a straightforward and rigorous theoretical treatment, in conformity with observations of atmospheric transmission, is not possible. A brief computation may illustrate this complexity.

The familiar exponential absorption law of Beer is

$$T_\lambda = e^{-\alpha w}.$$

Here T_λ is the transmission at wavelength λ, α is the absorption coefficient, and w is the amount of water vapor (or CO_2) penetrated. Beer's law only applies when the absorption coefficient α does not vary over the spectral band.

Since the infrared spectrum is almost completely covered by lines of about 0.1 cm.$^{-1}$ half-width, a precise accounting by means of this law would require one to calculate about 10 spectral regions per half-width, or about 100 regions per cm.$^{-1}$. The spectrum from 1 micron (or 10,000 cm.$^{-1}$) to say 10 microns (or 1000 cm.$^{-1}$) would therefore require calculations for many thousands of spectral intervals, each evaluated according to our present knowledge of dependence upon concentration, pressure, or path length. Further, we do not yet know all the lines of absorbing gases, nor their strengths.

An arbitrary assumption of one "average" absorption coefficient is

misleading, and it is inherent in the problem that the law of absorption is just as complex as the spectrum it is to describe. And yet, with certain compromises, for many applications, there should be some basically logical law which adequately combines the data of the diverse investigations and yields a useful common result. In order that the interested enginer can use such a law with facility and confidence, and even extrapolate to physical conditions not encompassed by prior observations, it must accurately describe the areas of mutual agreement among observers. Hence, although there is perhaps no *a priori* reason for supposing that the complicated phenomena of atmospheric transmission can be reduced to a simple empirical law, with only a few parameters, it was the purpose of this project to look for such a means of description.

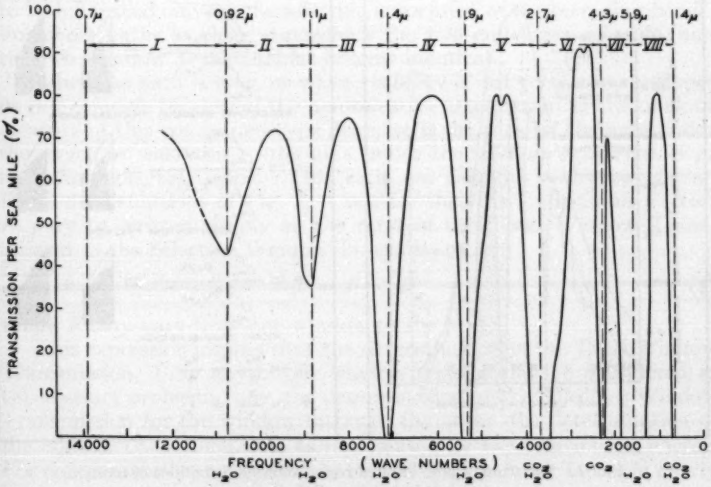

FIG. 1. Absolute infrared transmission of the atmosphere (adapted from Ref. 5).

GENERAL CONSIDERATIONS

The region considered in this report is displayed in Fig. 1, which is a smoothed *observed spectrum* (adapted from the absolute transmission measurements of Gebbie *et al.*).[3] This spectrum has been replotted on a frequency scale to demonstrate the band widths. The atmospheric windows, or regions of relatively high transmission contained between centers of adjacent strong bands, are identified for convenience with the Roman numerals I to VIII, and the window boundaries are so labelled, with appropriate wavelengths (which are repeated in Table I).

[3] For literature references mentioned, see the "Selected Bibliography" appended to this paper.

Primarily because of continuous attenuation due to haze, the spectral intensities in the centers of the windows are not the same as they would be if the optical path were evacuated. Fog, smoke, or even long paths of clear atmosphere all give rise to some continuous attenuation. By the simple construction of a smooth curve connecting the tops of the windows, to give an *envelope spectrum*, the continuous attenuation may be considered independently of the selective absorption. The attenuation below the envelope spectrum is presumed to be due almost completely to band absorption, whereas the depression of the envelope spectrum below the *vacuum spectrum* is presumed to be due essentially to the continuum-type attenuation.

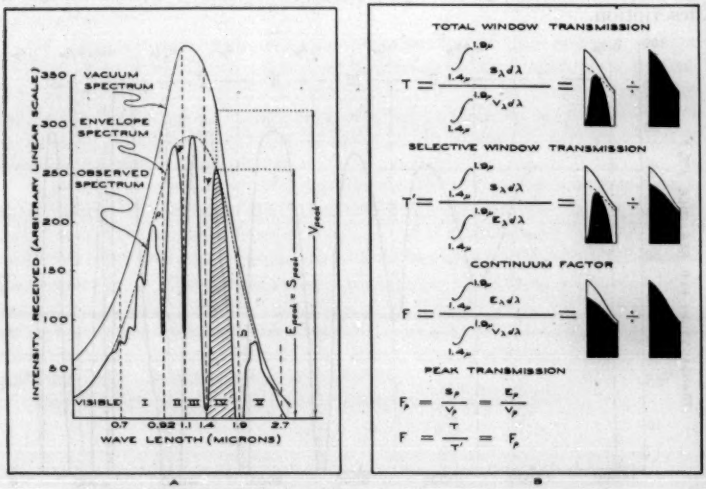

FIG. 2. Spectrum of near infrared region showing definitions of terms (adapted from Ref. 2).

This envelope is as significant when drawn on any published spectrum. Observers use various presentations of results—intensity, as well as transmission, may be plotted on a linear, or logarithmic, scale as ordinate, against wavelength, frequency, prism angle or grating angle as abscissa. Figure 1 is a graph of *absolute* transmission plotted against frequency.

Figure 2(A) shows a typical spectrum of the near infrared region with the windows labelled as before. The lower solid curve shows the spectrum actually observed, S, as a function of glass prism angles (although it is labelled in wavelength). The upper solid curve approximates the spectrum that would be received from the same source over

a vacuum path, V, and the intermediate dashed line is the envelope spectrum E, which separates the selective (or molecular) attenuation from the nonselective attenuation (due to continuous molecular absorption and particle scattering).

In Fig. 2(B) four quantities are defined in terms of Fig. 2(A). The *total window transmission*, T, is defined as the ratio of actual intensity received in a window to the intensity which would be received through a vacuum path in this spectral interval. In Fig. 2 this is represented for Window IV by the ratio of the shaded area below the observed spectrum to the area beneath the vacuum spectrum, both over the same spectral range of 1.4 microns to 1.9 microns. Similarly, the ratio of the shaded area to the area beneath the envelope spectrum or dashed line represents the *selective window transmission*, T', which we presume to be influenced only by the selective absorption of the bounding bands. For short paths in clear atmosphere the two envelopes coincide, and the two Window Transmissions become identical.

When the path is long, or when visibility is not good, the envelopes do not coincide because of the non-selective attenuation. The obvious measure of this continuum-type decrease is the ratio of the area under the envelope spectrum to the area under the vacuum spectrum, over the same spectral region. This ratio we call the *continuum factor*. From the definitions of Fig. 2, it is clear that the Continuum Factor, F, may be written simply as the ratio of the Total Window Transmission to the Selective Window Transmission, or

$$T = F \times T', \tag{1}$$

where F may vary from one window to the next.

This expression implies that the determination of the Total Window Transmission, T, in any of our window regions may be considered as two distinct problems: one, the determination of the Selective Window Transmission for the window interval; the other, the determination of the nonselective attenuation as represented by the Continuum Factor. For comparatively short paths on clear days, the latter factor is nearly one, but often it varies in a complicated manner. In any event, this separation allows comparison between field observations of different investigators, since it discriminates the effects of the more easily determined water vapor path from the indefinite effects of particles (which are not properly defined by visibility). In fact, the final justification of our separation procedure will be the mutual consistency of diverse observations, when treated by it.

SELECTIVE WINDOW TRANSMISSION

1. Water Dependence

We proceed now to deduce, on a logical basis, if not on a physical one, a formulation to describe the dependence on water path penetrated

of the Selective Window Transmission T'. The effects of continuum attenuation are given separate consideration later.

All observers are in qualitative agreement on one point: the radiation, in any given (relatively wide) spectral interval, absorbed by a given increment of water vapor dw, is proportional to the intensity of the incident source in that interval, I_0. We are concerned here with low spectral resolution, since the infrared spectrum out to 6 microns is divided into only 7 main natural divisions. Hence, we shall consider dI and I_0 as mean values for a whole window. Furthermore, the incremental absorption, dI, which is produced by any fixed dw, becomes relatively smaller as more and more water vapor is penetrated. These two statements may be expressed, to a first approximation, by the expression:

$$dI = -I_0 k\, dw/w. \tag{2}$$

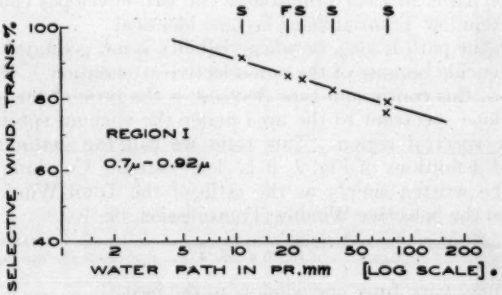

FIG. 3. Selective window transmission in Window I. Letters are identified with sources in Table I.

(We could just as logically have taken dI as proportional to some power of w other than -1, but that power has been found to fit the data well and the integral of Eq. 2 is conveniently plotted on semi-log paper.)

The increment of Selective Window Transmission, dT', is therefore

$$dT' = dI/I_0 = -k\, dw/w. \tag{3}$$

Integrating the differential equation (3) yields the Selective Window Transmission

$$T' = -k \log w + t_0. \tag{4}$$

Thus, when measured values of T' and w are plotted for any window on semi-log graph paper, the experimental points should fall on a straight line of slope $-k$, with an intercept t_0. It is difficult, and for the present purpose unnecessary, to assign direct physical meaning to the empirical constants k and t_0. We shall see that Eq. 4 represents the observations sufficiently well for engineering use and even for moderate extrapolation.

Figures 3 to 9 are graphs of T' *vs.* $\log w$ for Windows I to VII,

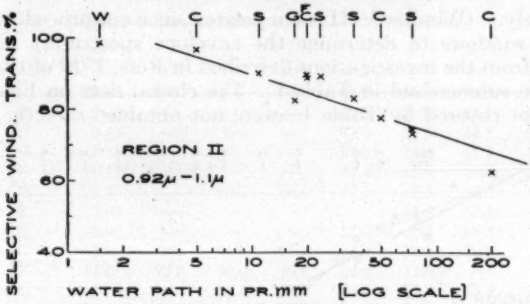

FIG. 4. Selective window transmission in Window II. Letters are identified with sources in Table I.

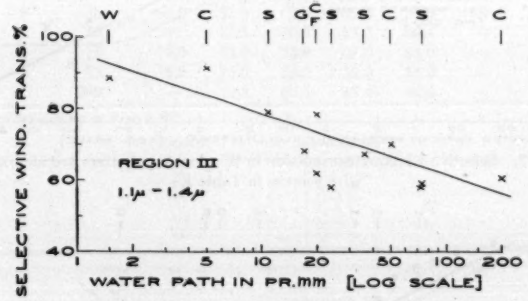

FIG. 5. Selective window transmission in Window III. Letters are identified with sources in Table I.

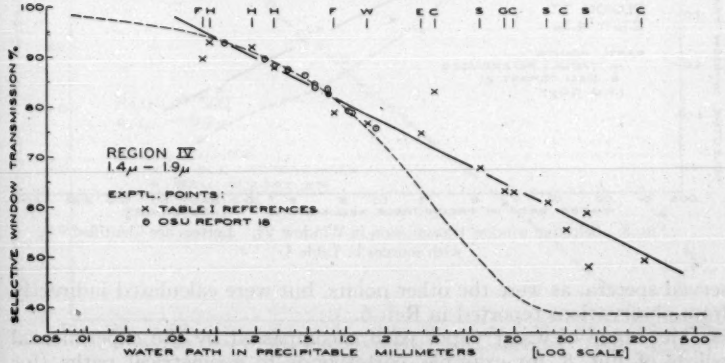

FIG. 6. Selective window transmission in Window IV. Letters are identified with sources in Table I.

respectively. (Window VIII is not treated since our procedure requires adjacent windows to determine the envelope spectrum.) The data, obtained from the investigations described in Refs. 1–10 of the Bibliography, are summarized in Table I. The circled dots on Figs. 6, 7, 8, and 9—not covered by Table I—were not obtained directly from ob-

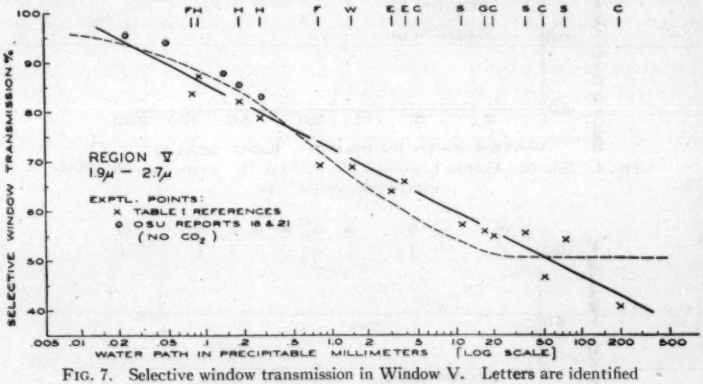
FH H H F W E E C S GC S C S C
SELECTIVE WINDOW TRANSMISSION %
REGION V
1.9μ – 2.7μ
EXPTL. POINTS:
X TABLE I REFERENCES
O OSU REPORTS 18 & 21
(NO CO2)
WATER PATH IN PRECIPITABLE MILLIMETERS [LOG SCALE]

FIG. 7. Selective window transmission in Window V. Letters are identified with sources in Table I.

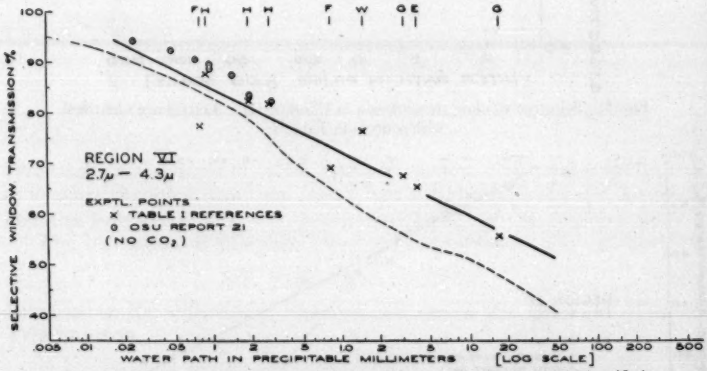
FH H H F W G E G
SELECTIVE WINDOW TRANSMISSION %
REGION VI
2.7μ – 4.3μ
EXPTL. POINTS
X TABLE I REFERENCES
O OSU REPORT 21
(NO CO2)
WATER PATH IN PRECIPITABLE MILLIMETERS [LOG SCALE]

FIG. 8. Selective window transmission in Window VI. Letters are identified with sources in Table I.

served spectra, as were the other points, but were calculated indirectly from observations reported in Ref. 6.

The range of water vapor path encompassed by the experimental points of Fig. 8, for example, corresponds to geometrical paths (for 70° F. and 70% relative humidity) extending from 1 to 20,000 yards.

TABLE I.—*Selective Window Transmission (per cent)*

Source	Code*	Water Path pr. mm.	I	II	III	IV	V	VI	VII	Ref. No.
Fowle	F	0.08	—	—	—	89.9	83.5	77.1	77.5	1
Howard	H	0.09	—	—	—	93.0	87.0	87.5	69.7	7
Howard	H	0.19	—	—	—	92.0	82.0	82.2	66.2	7
Howard	H	0.28	—	—	—	88.0	78.4	81.5	61.2	7
Fowle	F	0.82	—	—	—	79.6	68.9	69.0	52.4	1
Hettner	W	1.5	—	98.5	88.6	77.0	68.5	76.2	—	10
E & MacN	E	3.05	—	—	—	—	63.5	67.5	—	4
E & MacN	E	3.9	—	—	—	75.0	65.4	65.3	—	4
Fischer	C	5.0	—	96.6	91.6	83.3	71.1	—	—	3
Strong	S	11.0	91.0	90.0	79.0	68.0	57.0	—	—	2
Gebbie	G	17	—	82.6	68.0	63.4	55.8	55.6	25.0	5
Fischer	C	20	—	88.8	78.4	63.2	54.8	—	—	3
Fowle	F	20	86.0	89.0	62.0	—	—	—	—	1
Strong	S	24	85.0	89.0	58.0	—	—	—	—	9
Strong	S	36	82.0	83.0	67.0	61.0	55.0	—	—	2
Fischer	C	50	—	77.5	70.0	55.5	46.4	—	—	3
Strong	S	75	79.0	73.0	58.0	47.0	54.0	—	—	2
Strong	S	75	75.0	75.0	59.0	59.0	54.0	—	—	2
Fischer	C	200	—	62.8	60.1	49.4	40.6	—	—	3

* Reference designation on Figs. 3–9:

Window	Region Limits
I	0.7μ–0.92μ
II	0.92μ–1.1μ
III	1.1μ–1.4μ
IV	1.4μ–1.9μ
V	1.9μ–2.7μ
VI	2.7μ–4.3μ
VII	4.3μ–5.9μ

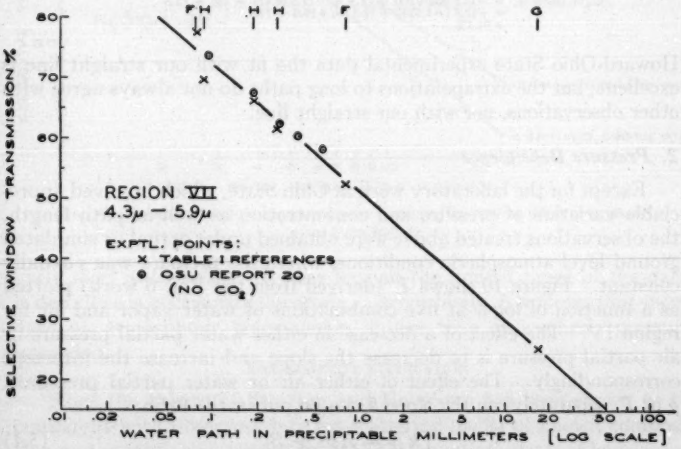

FIG. 9. Selective window transmission in Window VII. Letters are identified with sources in Table I.

The empirical constants associated with the solid straight lines on the graphs are displayed in Table II.

The average straight lines of these seven graphs show the validity of Eq. 4, while the points show the agreement of the observed data. It is at once apparent to what extent Eq. 4 can be used to describe the Selective Window Transmission as a function of the water vapor penetrated. The appropriateness of this empirical description of the observations would appear to warrant its use for prediction.

On Figs. 6, 7, and 8 a dashed curve is drawn. This represents the Selective Window Transmission as predicted for no-haze conditions from the recently published tables of Yates (which tables were derived from the data of Howard). It can be seen that in the region of the

TABLE II.—*Empirical Constants from Figs. 3–9*

$T' = -k \log w + t_0$

T' is Selective Window Transmission in per cent
w is Water path in precipitable millimeters

Window	Region	k	t_0	$T' = 100\%$ if w is less than
I	0.70 to 0.92μ	15.1	106.3	0.26 mm.
II	0.92 to 1.1	16.5	106.3	0.24
III	1.1 to 1.4	17.1	96.3	0.058
IV	1.4 to 1.9	13.1	81.0	0.036
V	1.9 to 2.7	13.1	72.5	0.008
VI	2.7 to 4.3	12.5	72.3	0.006
VII	4.3 to 5.9	21.2	51.2	0.005
VIII	5.9 to 14	(not treated)		

EXAMPLE: For window VI, and 0.03 precipitable millimeters of water

$$T' = -12.5 (\log 0.03) + 72.3 = -12.5 (8.477 - 10) + 72.3$$
$$= -12.5 (-1.523) + 72.3 = +19.0 + 72.3$$
$$= 91.3\%.$$

Howard-Ohio State experimental data the fit with our straight line is excellent; but the extrapolations to long paths do not always agree with other observations, nor with our straight line.

2. Pressure Dependence

Except for the laboratory work at Ohio State, which involved appreciable variation of pressure and concentration as well as path length, the observations treated above were obtained under actual or simulated ground level atmospheric conditions, where the pressure was virtually constant. Figure 10 shows T' (derived from the Ref. 6 work) plotted as a function of $\log w$ at five combinations of water vapor and air for region IV. The effect of a decrease in either water partial pressure or air partial pressure is to decrease the slope and increase the intercept correspondingly. The effect of either air or water partial pressures, p or P, separately, on the slope k can be put in the form:

$$[k]_P = Ap^\alpha + B \tag{5A}$$

$$[k]_p = DP^\beta + E. \tag{5B}$$

Here α is found to be nearly 1/8 for air, and β about 1/4 for water. In Figs. 3 to 9 the water is sufficiently dilute with respect to the air that we may take the water-water effect as negligible compared to the air-water effect. A combination of formulas 5*A* and 5*B* leads to unwieldy and inappropriate relations.

When the partial pressure of the water is small compared to that of the air, we find that the observed dependence on the pressure can be approximated by the expression:

$$T_1' = T_2' \times (P_2/P_1)^{1/8}. \tag{6}$$

Here T_2' is the Selective Window Transmission at the total pressure P_2. *One should keep in mind that this pressure dependence is based on rela-*

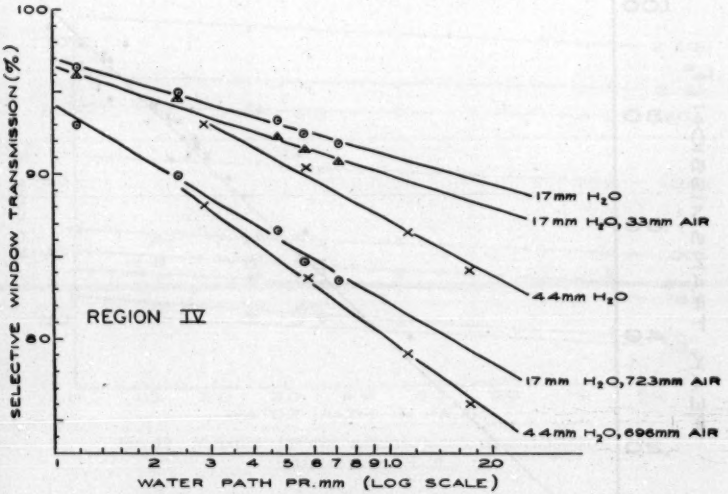

FIG. 10. Selective window transmission in Window IV, showing effect of pressure (data from Ref. 6).

tively short water paths. Its use beyond the limited range of the data it describes is necessarily uncertain. Accordingly, it appears that more data on atmospheric transmission at less than atmospheric pressure is needed.

NON-SELECTIVE ATTENUATION

Since the theory of Stratton and Houghton has been shown to be consistent with observed data for scattering media of known composition and particle size, prediction of the attenuating effects of haze or fog would presumably be possible. But, whereas the precipitable water

vapor in the path may be inferred accurately by simple humidity measurements, the composition and size of scattering particles is difficult to determine. One may use previous experimental results to predict the effects of non-selective attenuation in a particular instance, but this procedure is only as valid as the assumption that the haze, fog, etc.,

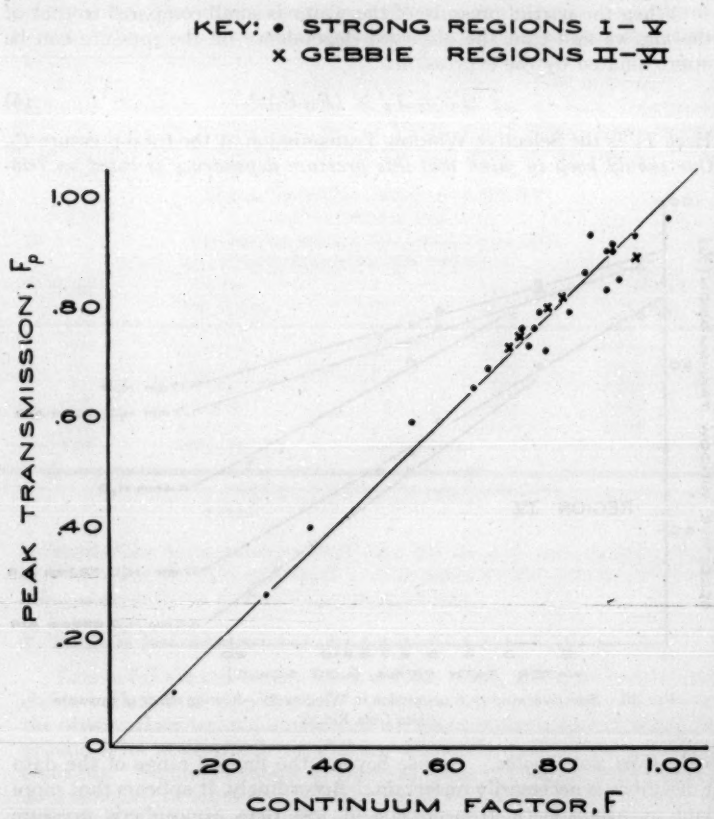

FIG. 11. Correlation of F_p with F (data from Refs. 2 and 5).

whose transmission is predicted is of the same nature as that previously observed and reported in the literature.

The Continuum Factor, F, defined in Fig. 2*B* is somewhat awkward and difficult to determine directly, but it is measured by the more conveniently determined monochromatic transmission at the *peak* of the

window. This quantity, which we call the *peak transmission*, F_p, is the ratio of the intensity of the observed spectrum at the peak of the window, S_p, to the intensity of the vacuum spectrum at this same wavelength, V_p. Thus, the Continuum Factor measures the effect of haze, fog, smoke, throughout the window while the Peak Transmission measures it only at the single, peak wavelength. These two descriptions should be equivalent if our window regions are sufficiently narrow (and it appears that they are).

Figure 11 shows a graph of the measured Peak Transmission as a function of the measured Continuum Factor, obtained from data of Strong and of Gebbie *et al.*, for six of the window regions. In these

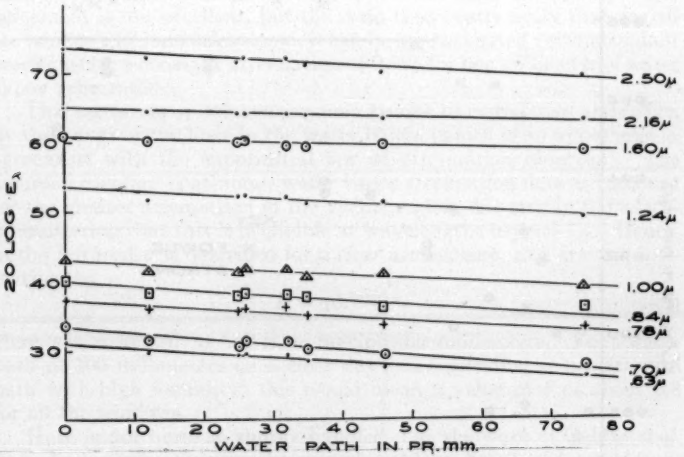

FIG. 12. Variation of E with water content in clear atmosphere (adapted from Ref. 2).

works, the vacuum spectrum appears to have been reliably determined by ingenious methods which are described in the original publications. The fact that the experimental points cluster about the straight line inclined at 45° to the axes indicates that the non-selective attenuation is described equally well by the Continuum Factor or by the Peak Transmission:

$$F = F_p = S_p/V_p. \tag{7}$$

Since the variation of the non-selective attenuation with wavelength is slow, the intensity of the envelope spectrum at any wavelength λ can be described by Beer's law:

$$E_\lambda = V_\lambda e^{-A_\lambda R}. \tag{8}$$

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Here R is the range and A_λ is a constant for a given atmospheric condition. In particular at the peak of a window, $E_p \equiv S_p$ and at this wavelength the observed spectral intensity follows exactly the same exponential law that the envelope spectrum does. Hence, if $\log S_p$ is plotted, as ordinate, against R, as abscissa, a straight line is obtained whose slope is $-A_p$, and the extrapolated intercept at $R = 0$ is $\log V_p$. Or, knowing two sets of corresponding values of R and S_p, one can solve for A_p and V_p.

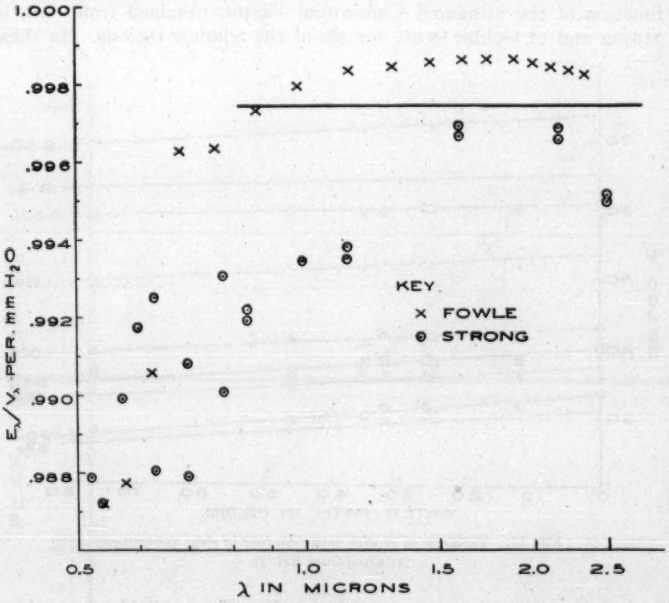

FIG. 13. Attenuation for one millimeter of water as a function of wavelength (from Refs. 1 and 2).

In this light we now consider two cases of non-selective attenuation separately:

1. Atmosphere very clear in the visible. In this case the non-selective attenuation in the infrared is due primarily to continuous absorption by water vapor in the optical path.

2. Hazy, foggy, misty, cloudy, or smoky atmosphere. Here both visible radiations and infrared radiations are appreciably attenuated by particles of water, of solutions, or of solids (whose radii are of the order of a micron).

In the first of these two cases the continuous absorption becomes of importance only for very long water paths. Figure 12 shows a graph of the logarithm of the transmitted intensity at the peaks of the windows (and at certain other wavelengths), as ordinate, and the water vapor penetrated as abscissa. This graph shows the dependence of non-selective attenuation on water for the clear atmosphere. The fact that the lines obtained from these experimental data are straight confirms the appropriateness of an exponential description such as Eq. 8. The slopes of these lines provide values of A_λ which determine the value of the ratio E_λ/V_λ for one millimeter of water vapor penetrated.

Figure 13 shows the wavelength variation of this ratio of E_λ/V_λ taken from these data (of Strong) and other data (of Fowle). The agreement is not excellent, but the ratio is so nearly unity that, for all the windows beyond one micron, it can be approximated (with abundant precision) by a constant attenuation of 0.998 for one millimeter of water vapor penetrated.

This water vapor attenuation may be due to cumulative absorption by the *wings* of the lines in the water bands (which is an hypothesis in agreement with the exponential law of attenuation observed). The assumed constant continuous water vapor attenuation does not account for the greater attenuation in the visible region, due largely to molecular scattering; but this is negligible at wavelengths beyond 1μ. Hence, in the infrared F is described for a *clear* atmosphere, to a first approximation, by

$$F = (0.998)^w. \tag{9}$$

Here w is expressed, as before, in precipitable millimeters. For a water path of 100 millimeters on a clear day (corresponding to a 6000 yard path with high humidity) this would mean a value of F of about 0.8 for all the windows.

Haze is not quite so simply handled, but the work of Gebbie *et al.* gives both verification for the exponential law, Eq. 8, and actual non-selective particle attenuation measurements. The fact that Gebbie found a functional relationship between visible transmission and infrared transmission (for constant water content) indicates that at his geographical location the haze observed was reproducible in its effect, if not precisely in its composition. Figure 14 shows experimental data of Strong (corrected to zero water by Eq. 8) and of Gebbie *et al.* Both are fitted to the same visible transmission. Particularly for light haze, the results are much the same, which implies that the haze effect is similar at both locations (both were paths over coastal sea water).

The effects of attenuation due to smoke, fog and clouds are not so easily treated, since available data do not show quantitative agreement because of the different experimental conditions and practices. In particular, observations of total radiation, or radiation over an extended spectral interval, will be complicated by concurrent selective absorption effects.

Mist or very weak fog is much the same as haze, and hence somewhat less attenuation is found at the longer wavelengths than in the visible region. Most artificial smokes have particles no larger than about 2 microns in diameter, and as a consequence even in the spectral region from 1 to 6 microns better transmission is found for longer wavelengths. For thick fogs and clouds, the larger particles render this whole spectral interval opaque, and only infrared of still longer wavelengths can penetrate fog or clouds.

The Gebbie observations are the best available on actual haze transmission, although it is yet to be established that the composition of

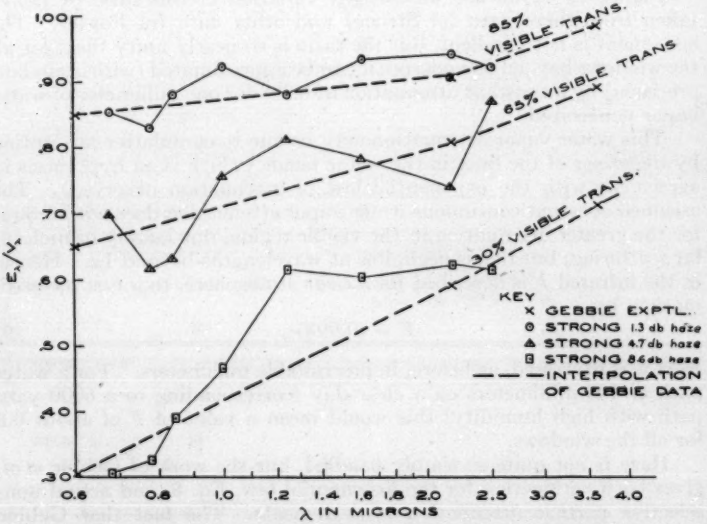

FIG. 14. Wavelength dependence of haze attenuation (from Refs. 2 and 5).

haze is universally the same. Yates gives a graph which generalizes Gebbie's results so that one may predict the scattering at ranges other than Gebbie's 2000-yard path.

In resumé: when the observations of diverse observers are lumped and considered window by window, the selective band absorption can be accounted for by Eq. 4 and the continuous attenuation approximated by Eq. 8. The latter is made up of two parts, that due to water vapor and that due to particles. The first is a small effect and can be predicted by Eq. 9. The continuous attenuation due to particles can be estimated from the visibility after Yates' generalization of the observations of Gebbie *et al.*, but this assumes a universal particle size distri-

bution and composition. Strong's observations across a coastal water path at Boston Harbor are in rough agreement with the Gebbie *et al.* observations across a similar path (at Tantallon, near Edinburgh, Scotland) and thus gives this assumption some validity.

More work needs to be done on particle attenuation, since predictions based on existing data are still inadequate for all of the needs of meteorology.

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This paper summarizes pioneer work at Mount Wilson begun in 1908 on the infrared spectrum of water vapor, involving both laboratory cell and out-of-door absorbing paths. It describes atmospheric transmission studies made before 1917 for the entire infrared region, especially the work of Fowle. Tables of non-selective attenuation for $\lambda < 2\mu$ are included, these separating the effect of air scattering from that due to water in presumably clear atmosphere. Spectra of the near infrared region are also displayed.

A rather detailed discussion of some of the experimental techniques, difficulties, and corrections found necessary comprise the bulk of the article. With the sun as source, the transmission of the vertical atmosphere was observed, and with a Nernst glower as source laboratory paths of 422 and 38 ft. were traversed by the radiation. (68 pages.)

(2) J. D. STRONG, OSRD Report, 5986, Nov. 30, 1945 (Harvard). "Atmospheric Attenuation of Infrared Radiations."

The 5000-yd. horizontal over-water path in Boston Harbor offered opportunities for observations under haze and high humidity conditions as well as through very clear and dry atmosphere. Solar spectra at different zenith angles (including near-horizon) for the infrared region were used to determine the variation of transmission at the tops of the windows with air mass and hence with water path. For $\lambda < 3\mu$, the intensity at wavelength 2.16μ varied least of all with haze, and the vacuum spectrum was inferred by fitting the envelope *shape* received over a 14-yd. path at this point. Thus absolute measurements were approximated without reference to visible wavelengths directly.

Solar bolograms were taken for $\lambda \leq 12\mu$, and the attenuating effects of various smokes at different wavelengths were compared. Transmission measurements in the region $\lambda < 2.7\mu$ show water and haze dependence separately.

An incidental result is a comparison of observed solar intensities at different zenith angles both with the formulas of Bemporad and with the secant of the zenith angle. Sources used were the sun, tungsten lamp, and a 700° C. heater. (25 pages.)

(3) HEINZ FISCHER, USAF Technical Report F-TR-2104-ND, Oct. 1946 (Wright Field). "The Influence of the Spectral Transmission of Optics and Atmosphere on the Sensitivity of Infrared Detectors."

This report is primarily concerned with the evaluation of infrared detectors. However, it contains atmospheric transmission spectra (presumably taken by Czerny) used in the present paper to determine Selective Window Transmission. The spectra are for path lengths of 1000 and 10,000 meters, over the spectral interval 0.9 to 3.4 microns. In general the textual material concerning atmospheric transmission is confined to brief statements.

The keynote of this report is the definition of "effective radiation" which is, in analogy to visual brightness in photometry, a composite integral over some spectral interval of the product of the spectral selectivities of the source, optics, atmosphere, and detector. Values of this integral for various conditions and detecting cells are compared. Much of the data are of wartime German origin. (23 pages.)

(4) A. ELLIOTT, G. G. MACNEICE, AND E. A. JONES, A.R.L./R. 2/E-600 Feb. 1946 (Admiralty Research Laboratory). See also *Nature*, Vol. 161, p. 516 (1948). "The Transmission of the Atmosphere in the Near Infrared."

This report resulted from an interest in the atmospheric windows at 2.2μ and 3.7μ (our V and VI). The conclusion was that these windows were approximately equally clear, and that the 4.3μ CO_2 band provided the long wavelength cut-off of the latter.

A Nernst glower was used as source, over a 600-yd. horizontal open air path. The location at Tantallon, off the coast of Scotland, provided varied visibility conditions. (4 pages.)

(5) H. A. GEBBIE, W. R. HARDING, C. HILSUM, A. W. PRYCE, AND V. ROBERTS, *Proc. Roy. Soc.*, Vol. A. 206, p. 87 (1951). Also ARL/R. 4/E-600 (Dec. 1919). "Atmospheric Transmission in the 1–14 Micron Region."

In this excellent paper the measured absolute transmission of the atmosphere per sea mile in the 1–14μ region is given for a water path of 17 precipitable millimeters per sea mile, and certain known visibility conditions. The horizontal paths of 2264 and 4478 yards used were believed to give sufficient attenuation to warrant extrapolation to longer paths.

Comparisons of the transmission at 2.18μ, 3.61μ, 10.01μ, and 11.48μ with that at 0.61μ (where absolute measurements of the transmission could be made photometrically) were used to obtain the absolute transmission at these four chosen infrared wavelengths. These were then used as standards for the adjoining infrared wavelengths. For these four reference wavelengths (in the peaks of windows V, VI, and VIII) it was found possible to correlate the infrared transmission with the 0.61μ transmission or "visibility," the latter being a measure of the haze present. Tables and graphs of the average values of the transmission per sea mile are presented for the five reference wavelengths under various visibility conditions. The results of the measurements for the two different path lengths used are compatible with an exponential law of attenuation for these peak wavelengths.

Variation of the transmission with humidity was found to be considerably less than that with visibility, and the 10.01 and 11.48μ measurements indicated an exponential variation of transmission with water path at these reference (peak) wavelengths. At the shorter reference wavelengths there was relatively weaker dependence on the water path, although it was observed that "there is a tendency for higher water-content to cause lower transmission."

Also included are the results of calculations which relate the observed variation of infrared transmission with visibility to the scattering of the radiation by salt nuclei. These are assumed to increase in size and change refractive index as the humidity increases. The size and concentration distribution is in agreement with what has already been reported in the literature. The calculated most-frequently-occurring droplet radius is less than one-half micron even for the greatest amounts of haze observed, and is smaller still for better visibility conditions. (20 pages.)

(6) R. M. CHAPMAN, J. N. HOWARD, AND V. A. MILLER, Ohio State University Research Foundation. Reports No. 18, June 30, 1949; 20, Oct. 15, 1949; 21, Dec. 15, 1949. Contract W-44-099-eng-400. The Engineer Research and Development Laboratories, Fort Belvoir, Va. "Atmospheric Transmission of Infrared." See also *J.O.S.A.*, Vol. 43, p. 423 (1952).

These experimental reports contain a brief discussion of the theory of absorption by a single line, by a group of lines, and by a band. Semi-empirical formulae are developed which represent well the observed absorption for five water bands, a group of water lines, a single water line, and two carbon dioxide bands (1.35μ, 1.85μ, 2.7μ, 3.2μ, 6.3μ bands of H_2O and 2.7μ, 4.3μ bands of CO_2). The average fractional absorption, A, in each of the water bands was represented by an error function:

$$A_w = \frac{2}{\sqrt{\pi}} \int_0^t e^{-t^2}\, dt$$

V.2
195

where

$$t = \frac{\sqrt{\pi}}{2}\beta[w]^{\frac{1}{2}}\left(\frac{P_t + P_p}{760}\right)^{\frac{1}{2}}.$$

The carbon dioxide band absorption did not follow this, and instead a simple exponential relation was used:

$$A_s = 1 - e^{-\beta[w]^n\left(\frac{P_t+P_p}{760}\right)}.$$

Here w is the amount of absorber (in pr. cm. of H_2O or Atmo-cm. of CO_2); P_p and P_t are the partial pressure of the absorber and the total pressure, respectively; β is a different constant for each band; and n is a fraction, different for each CO_2 band. Values of β and n are given for each of the bands studied.

The path lengths used ranged from 4 to 48 meters, with pressures extending from a few millimeters of mercury to atmospheric pressure. Water paths used ranged from 0.002 to 0.2 pr. cm.; CO_2 paths, from 5 to 105 atmo-cm.

Many graphs of per cent absorption *vs.* frequency for each band are included, for different combinations of pressure, absorber concentration, and path lengths, all at constant temperature. (59 pages of text.)

(7) J. N. HOWARD, Ohio State Univ. Res. Foundation, Report No. 1, March 31, 1950. Contract D.A. 44-009-eng-12. The Eng. Res. & Development Laboratories, Ft. Belvoir, Va. "The Absorption of Near Infrared Blackbody Radiation by Atmospheric Carbon Dioxide and Water Vapor."

The empirical formulae developed in Ref. 6 are here used to calculate the "tota, energy transmitted through the atmosphere in the spectral regions 0.7 to 3μ, 0.7 to 6μl 0.7 to 12μ, for blackbody sources at various temperatures, as a function of the water vapor density in the air." Extrapolating the Ref. 6 data, the author gives graphs predicting, for various blackbody sources (at temperatures between 30 and 3000° C.), the total energy per unit area per unit time transmitted in each of these three spectral intervals. The paths cover the range from 1 to 10,000 yd., for clear atmosphere at different humidities.

Composite spectra taken from data of Ref. 6 are included, and these were used to calculate the Selective Window Transmission for our Table I.

The author points out the failure of the Beer-Lambert Law for total absorption in the 1–6μ region. The per cent transmission of each of the CO_2 and water bands is plotted also as a function of geometrical path (at different absorber concentrations. (14 pages.)

(8) H. YATES, NRL Report 3858, Sept. 10, 1951. Naval Research Laboratory, Washington, D. C. "Total Transmission of the Atmosphere in the Near-Infrared." See also E. O. HULBERT, NRL Report 2348, 22 July 1944.

The predictions from Ref. 7 of total transmission in clear atmosphere are combined in this report with the observations from Ref. 5 of the effect of haze on infrared transmission. The result is the prediction of total atmospheric transmission between 0.7 and 12μ as a function of source temperature, geometrical path, water content of the air, and visibility at 0.61μ, that is, haze. Graphs display this prediction as function of geometrical path from 1 to 20,000 yd. The spectral region 0.7–12μ is divided into twelve sub-regions. Tables show the fraction of blackbody radiation transmitted in each of these sub-regions, for different geometrical paths, blackbody temperatures, and visibility conditions.

The paper also contains a graph showing E_λ *vs.* λ, interpolated from the data of Ref. 5 for various haze conditions (similar to our Fig. 14). Two useful conversion graphs are also included: one, relating water vapor concentration to relative humidity or to precipitable centimeters of water per 1000 yd.; the other, using Beer's Law to

relate the per cent transmission per 2000 yd. to the per cent transmission per 100, 1000, 10,000, and 20,000 yd. (6 pages of text.)

(9) J. STRONG, JOUR. FRANKLIN INST., Vol. 231, p. 121 (1941). "On a New Method of Measuring the Mean Height of the Ozone in the Atmosphere."

This paper was used as a reference because of an absorption spectrum of the atmosphere from which the Selective Window Transmission could be obtained. The sun served as source, and the path was approximately the vertical atmosphere. (34 pages.)

(10) G. HETTNER, *Annalen d. Physik*, Vol. 55, p. 476 (1918). "Über das Ultrarote Absorptionsspecktrum des Wasserdampfes."

This paper deals with the low resolution spectrum of steam in the infrared. In the wavelength region from which the Selective Window Transmission was obtained for our Table I, the geometrical path length was 109 cm, and the mean temperature was 127° C. (20 pages.)

(11) H. GAERTNER, NAVORD Report 429 (1947), U. S. Gov. Printing Office. "The Transmission of Infrared in Cloudy Atmosphere."

This is a translation of a 1942 German summary report. The theory of scattering is briefly described, and experimental work on fog transmission in both visible and infrared spectral regions is extensively reviewed. The conclusion is drawn that the experimental results agree well with the theory.

Brief mention is made of selective absorption and of attenuation in rain.

A bibliography of 70 items is included. (65 pages.)

(12) C. HILSUM, ARL/R. 3/E-600, December 1948. Admiralty Res. Lab. Teddington, Middlesex, Eng. "Atmospheric Attenuation of Infrared and Visible Radiation."

This report contains a complete, indexed bibliography of over a thousand references to the literature on the subject of atmospheric transmission prior to 1949. The text serves as a guide to the bibliography, and the causes and effects of scattering comprise the bulk of both.

". . . a short discussion is given of the laws of absorption and the effects of pressure and temperature. The theory of scattering by small particles is mentioned; in the visible the attenuation is caused almost completely by scattering, and the visual range can be evaluated in terms of the number and size of the particles in the atmosphere. There are two main types of particles, the combustion nuclei and the sea-salt nuclei.

"When condensation takes place on the nuclei, a fog is formed. The transmission of radiation through fogs is determined mainly by the size of the fog droplets, and several methods of measuring the droplets are given. The methods are discussed and the results summarized. Comparisons have been made in fogs of the visible transmission and the infra-red transmission, but the various results are contradictory. In artificially produced fogs and chemical smokes the observed variation of attenuation with wavelength agrees with that predicted by theory. A brief mention is made of transmission in rain and snow." (36 pages of text.)

THE REACTION OF FORMALDEHYDE WITH D-THREONINE *

BY

D. C. CARPENTER [1]

The combining of formaldehyde with D-threonine as its sodium salt has been studied by the polariscopic method. A compound of the AF type and one of the A_2F_3 type have been found. The equilibrium constant of the latter reaction has been calculated. Neither compound could be isolated from solution.

INTRODUCTION

The reaction between formaldehyde and the amino acids has been studied by numerous workers. The reactions have been followed by measurements of the unreacted amino groups, hydrogen-ion concentration and optical rotation and various constants evolved (1).[2] Many of these authors worked at pH's where the amino acid existed partly as anion and partly zwitter-ion, which made interpretation of data difficult; furthermore, workers have seldom recognized that the reactions are comparatively slow and have not allowed sufficient time for attaining equilibrium.

A study of the pK values for the various amino acids showed that in general if an equivalent of base was present in the solution, practically all of the amino acid was present as anion and the zwitter-ion eliminated. If the above was not accomplished by the added base, the first few additions of aldehyde changed the anion:zwitter-ion equilibrium so that the latter ion was reduced to zero concentration. All of the amino acid was present thereby in one form and calculation of results was simplified. Measurement of the rotation in sealed polariscope tubes over long periods of time at constant temperature was an easy matter. The chief objection resides in the matter that a pure optical isomer is required.

Studies in this laboratory on the binding of aldehyde by amino acids have dealt with asparagine, aspartic and glutamic acids and histidine. All of these form 1:1 mole compounds (AF) with formaldehyde. These compounds react further with increasing amounts of formaldehyde to give compounds which cannot be isolated without decomposition. With the first three amino acids, the higher compound is of the AF_2 type but in the case of histidine the second compound is of the A_2F_3 type.

This paper reports the reaction of formaldehyde with D-threonine

* Published as Journal Paper No. 880 of the New York State Experiment Station.

[1] New York State Experiment Station, Geneva, N. Y. [EDITOR'S NOTE: Dr. Carpenter died unexpectedly, in January of this year.]

[2] The boldface numbers in parentheses refer to the references appended in this paper.

to form a 1:1 mole compound and the further reaction of two moles of this compound with one mole of additional aldehyde. The latter reaction is reversible and the reaction product unstable and its existence in solution is shown by the equilibrium constant which is some three and one-half times that of the corresponding histidine reaction.

PREPARATION OF MATERIALS

D-threonine.—A commercial sample of DL-threonine was converted to the *p*-nitrobenzoyl derivative and resolved as the respective brucine salts from warm methanol, essentially as described by Zambito *et al.* (2).

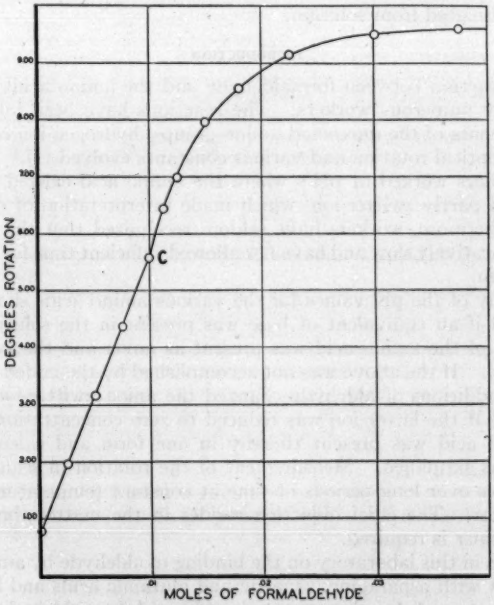


FIG. 1. Effect of added formaldehyde on angular rotation (2 dm.) of 0.01 mole of D-threonine containing one equivalent of sodium hydroxide in 25 ml. of solution.

After removal of brucine and the *p*-nitrobenzoyl group, D-threonine was recrystallized twice from absolute ethanol and dried in vacuo. Amino acid nitrogen determination (Van Slyke) gave 11.70 per cent amino nitrogen (calcd. 11.76 per cent). Paper chromatography showed that only one substance was present which had R_F values of 0.49 (phenol-water) and 0.16 (pyridine-isoamyl alcohol) (3) showing the absence of *p*-nitrobenzoic acid. An aqueous solution gave $[\alpha]_D^{20°} + 28.37°$ ($C = 5$) as compared with Zambito's value $[\alpha]_D^{26°} + 27.8°$ ($C = 5$).

Formaldehyde.—A very pure concentrated formaldehyde solution was brought to exactly pH 7.0, measured against the glass electrode, by the addition of sodium hydroxide solution, and the aldehyde content of this stock solution determined by the dimedon method (4).

EXPERIMENTAL

Into each of a series of 25-ml. volumetric flasks, exactly 0.01 mole of D-threonine was weighed out, 0.01 mole of standardized sodium hydroxide solution added and the mixture shaken until the amino acid was dissolved. Measured amounts of the stock formaldehyde solution

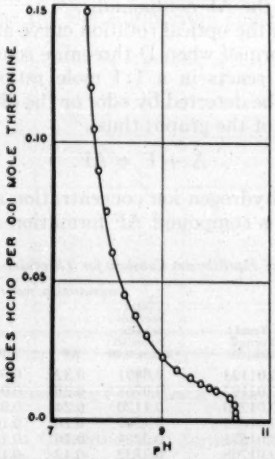


FIG. 2. Effect of added formaldehyde on pH of 0.01 mole of D-threonine containing one equivalent of sodium hydroxide in 25 ml. of solution.

were added from a microburet to each flask and the volume of each solution made up to the 25-ml. mark with water and well shaken. Part of each solution was placed in a 2-dm. polarizing tube and these tubes and the flasks containing each reserve solution were placed in a constant temperature bath at 20°. The rotation was read periodically in the polariscope with the sodium arc as light source. Equilibrium was reached in a few days with the more dilute solutions but in solutions containing more aldehyde than 0.01 mole, upwards of thirty days were required. After reaching equilibrium, the hydrogen-ion concentration of each reserve solution was measured with the glass electrode. The optical rotation data are plotted in Fig. 1 and the hydrogen ion data in Fig. 2.

DISCUSSION

The dissociation constants for threonine have not been established but we can assume that they are close to those of serine, for which $pK_a = 2.21$ and $pK_b = 9.15$ (5). Constructing the dissociation curves for anion and zwitter-ion for a 0.40-molar threonine solution shows that the threonine containing an equivalent of NaOH exists as about 90 per cent anion and 10 per cent zwitter-ion when the AF reaction begins with zero added aldehyde. As anion reacts with aldehyde forming the AF compound and is thereby removed from the zwitter-ion : anion equilibrium, zwitter-ion is converted to anion to maintain the ionic equilibrium until the zwitter-ion content is zero and all of the anion has reacted with aldehyde giving the AF compound.

From the break in the optical rotation curve at *C* (Fig. 1) it is clear that a compound is formed when D-threonine containing an equivalent of sodium hydroxide reacts in a 1:1 mole ratio with formaldehyde. No free aldehyde can be detected by odor or the dimedon reaction along the straight line part of the graph; thus

$$A + F = AF. \qquad (1)$$

The rapid change in hydrogen-ion concentration at the 1:1 mole ratio point (Fig. 2) confirms compound AF formation at this point.

TABLE I.—*Calculation of Equilibrium Constant for Threonine-Formaldehyde Reaction*

		Concentration, moles per liter					
Completion of Reaction	Rotation Degrees	Total Formaldehyde	Total Formaldehyde Times 40 Minus 0.40	AF	A_2F_2	Free Formaldehyde	*k*
0.20	+6.410	0.01123	0.0491	0.32	0.04	0.0091	0.0232
0.30	6.815	0.01192	0.0768	0.28	0.06	0.0168	0.0232
0.40	7.220	0.01280	0.1120	0.24	0.08	0.0320	0.0230
0.50	7.625	0.01395	0.1580	0.20	0.10	0.0580	0.0232
0.60	8.030	0.01571	0.2284	0.16	0.12	0.1085	0.0231
0.70	8.435	0.01708	0.2832	0.12	0.14	0.1432	0.0147
0.80	8.840	0.01965	0.3860	0.08	0.16	0.2260	0.0090
						Mean (5)	0.02314

In the curved portion of Fig. 1, all of the experimental solutions contain unreacted aldehyde and hence the reaction is of the equilibrium type and the determination of the equilibrium constant should be a criterion of the ratios of the substances undergoing reaction. Table I has been constructed for the reversible reaction

$$2\,AF + F \rightleftharpoons A_2F_2 \qquad (2)$$

the concentration of the free aldehyde, AF compound and the reaction product in moles per liter being read off the graph for a series of points, assuming that the angular rotation of each optically active substance is a measure of its respective concentration.

Computations were made of the equilibrium constant of the second reaction in which $C^2_{AF} \times C_F / C_{A_2F_2} = k$. The average of the first five closely agreeing values of *k* for this reaction is 0.02314 and represents

quite a range of aldehyde concentration and about two-thirds complete conversion to the A_2F_3 compound. The k value decreases as great excesses of aldehyde are employed, probably due to failure of the mass law as is common at high concentrations.

The equilibrium constant may be evaluated also by the graphical method in which the logarithmic form of the equilibrium equation is used and log AF^2/A_2F_3 is plotted against log F as shown in Fig. 3 and the value of log k read at the zero intercept. This method gives the value $k = 0.0234$, in good agreement with the treatment above.

The angular rotation of the AF compound was $+5.60°$ and that for the A_2F_3 compound was $+9.65°$. Reversible reactions other than (2) gave no constant value for the equilibrium constant and hence were eliminated.

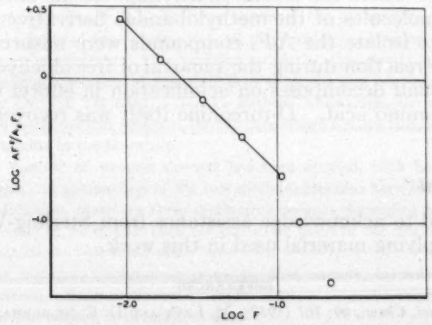
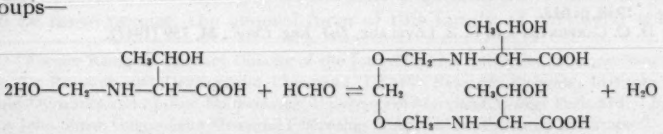
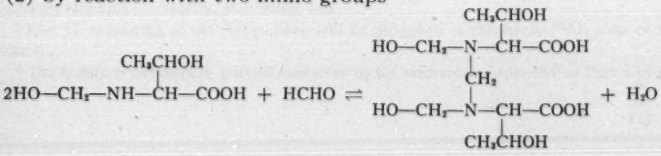


FIG. 3. Graph of log AF^2/A_2F_3 against log F for unstable threonine-formaldehyde compound.

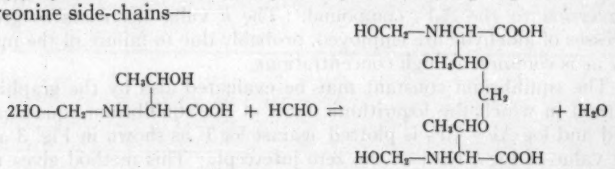
The AF compound of threonine and formaldehyde is no doubt the usual methylol-amino type of derivative commonly encountered in this reaction of amino acids. The second mole of aldehyde must link together two molecules of the 1:1 mole compound. This could be accomplished on the main chain (1) by reaction with two methylol-amino groups—



or (2) by reaction with two imino groups—



or (3) by reaction of the aldehyde to join the OH groups of the two threonine side-chains—



In the case of proteins we have pointed out (6) that in two parallel peptide chains the intermolecular distances and bond angles are correct for hemiacetal formation and bridging between two side-chains containing the OH group by a —O—CH_2—O-bridge. At present it cannot be certain as to where the second aldehyde molecule effects the union between two molecules of the methylol-amino derivative of threonine.

Attempts to isolate the A_2F_3 compounds were unsuccessful due to reversal of the reaction during the removal of free aldehyde. Further, the AF compound decomposes on acidification in efforts to obtain the free methylol-amino acid. D-threonine itself was recovered as shown by its rotation.

Acknowledgment

We are glad to acknowledge assistance from Sterling-Winthrop Institute for supplying material used in this work.

V.2
195

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(2) A. L. ZAMBITO, W. L. PERETZ AND E. E. HOWE, *ibid.*, **71**, 2541 (1949).

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ASYMPTOTIC EXPANSION OF THE WHITTAKER'S FUNCTION $W_{k,m}(z)$ FOR LARGE VALUES OF k, m, z

BY

CHIEH-CHIEN CHANG,[1] BOA-TEH CHU[2] AND VIVIAN O'BRIEN[2]

Part I[3]

SUMMARY

The present paper investigates some special cases of the asymptotic behavior of the Whittaker function, $W_{k,m}^{(n)}(z)$ for large values of the parameters (k, m) and the independent variable of the function (z), all of which depend on an eigenvalue (n) which increases indefinitely. This problem arises in approximating to the third order the hodograph solution of transonic flow as shown in a previous report of the senior author (NACA TN 2582). To the authors' knowledge, the asymptotic behavior of this mathematical function, for the case where k, z are real, m is imaginary, is not yet available in the literature.

Debye's method of steepest descent has been applied, with Langer's method used as a check. A comparison of the two methods has also been included. In the Whittaker's function, there are three distinctive regimes depending on whether z is larger than, equal to, or smaller than the transition point of the original Whittaker's differential equation. All three cases have been investigated, corresponding to the three types of hodograph solution in transonic flow, namely, (a) subsonic, (b) sonic and (c) supersonic. Some numerical examples of the asymptotic solutions have been calculated for comparison with exact values.

INTRODUCTION

In the problem of transonic flow using the hodograph method, a family of particular solutions of the third order approximation to Chaplygin's second equation can be constructed by the method of separation of variables. These particular solutions are found to be the product of the Whittaker's function and trigonometric functions (1).[4] To be more precise, the general form of this family of particular solu-

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[2] Research Assistant and Research Staff Assistant, respectively, Aeronautics Department, The Johns Hopkins University, Baltimore, Md.

[3] Part II, consisting of the Appendices, will be published in the April, 1953, issue of this JOURNAL.

[4] The boldface numbers in parentheses refer to the references appended to Part I of this paper.

tions is given by $W_{k,m}(z) \sin n\theta$ where

$$k = -\left(\frac{c-2b}{2c^2}\sqrt{\frac{a}{b}}\right)n + k_1$$

$$m = \left[\frac{1}{4} - \frac{n^2a(c-b)}{c^4}\right]^{\frac{1}{2}}, \quad (n > 1)$$

$$z = 2n\frac{\sqrt{ab}}{c^2}(1 + c\sigma).$$

θ = angle of the velocity vector in the physical plane. In these equations, a, b, and c are known positive constants which have been chosen to give the most optimistic approximation to exact gas properties. For the problem in Ref. 1, $a = 9.40$, $b = 1.61$, $c = 3.89$ so that $a > c > 2b$ and $c = \sqrt{ab}$. Hence m is real for $n = 1$ but becomes imaginary for $n \geq 2$. k_1 in the first equation is an arbitrary real constant introduced to make the analysis more general. In the expression of z, σ is a monotonic decreasing function of Mach number.[5] $\sigma > 0$ corresponds to subsonic flow ($M \to 0$, $\sigma \to \infty$); $\sigma = 0$ corresponds to sonic flow; $\sigma < 0$ corresponds to supersonic flow $\left(M \to \infty, \sigma \to -\frac{1}{c}\right)$. n is an eigenvalue. To indicate that k, m, and z are functions of the parameter n, we shall hereafter write $W^{(n)}_{k,m}(z)$ instead of $W_{k,m}(z)$.

Since the complex integral representation of the Whittaker function is well known, we shall obtain the asymptotic expansion by means of Debye's method of steepest descent. The technique is not new; however, in our case the analysis is rather involved. In order to familiarize the engineering scientist with this method, a detailed account of the investigation is given here so that similar investigations on other functions can be made without too much trouble.

As a check on the work, the analysis was also carried out by Langer's method (5) as shown in Appendix B.

This investigation was under the sponsorship of the National Advisory Committee for Aeronautics and was conducted at the Department of Aeronautics, The Johns Hopkins University.

ANALYSIS

The integral representation for the Whittaker function (2) is

$$W^{(n)}_{k,m}(z) = -\frac{1}{2\pi i}\Gamma(k + \tfrac{1}{2} - m)e^{-\frac{1}{2}z}z^k \times \int_{\infty}^{(0+)}(-t)^{-k-\frac{1}{2}+m}\left(1 + \frac{t}{z}\right)^{k-\frac{1}{2}+m}e^{-t}\,dt. \quad (1)$$

[5] In Ref. 1, $\sigma = \int_q^{a^*}\frac{\rho_0}{\rho}\frac{dq}{q}$. Here q is the velocity magnitude of gas flow, a^* is the sound velocity and $\frac{\rho}{\rho_0}$ is the ratio of the local density to stagnation density.

In our case

$$k = -n\frac{c-2b}{2c^2}\sqrt{\frac{a}{b}} + k_1 = -nk_0 + k_1 \tag{2a}$$

$$m = \left[\frac{1}{4} - \frac{n^2a(c-b)}{c^4}\right]^{\frac{1}{2}} = inm_0 + O\left(\frac{1}{n}\right), \quad (n > 1) \tag{2b}$$

$$z = \frac{2n}{c^2}\sqrt{ab}(1 + c\sigma) = nz_0, \tag{2c}$$

where z_0, k_0, m_0 are real positive numbers. In the above, k_1 is an arbitrary real constant which may, of course, be zero.

The notation $\int_\infty^{(0+)}$ means that the path of integration starts at infinity on the real axis, encircles the origin in the positive direction and returns to the starting point. By Cauchy's theorem all such contours that satisfy these requirements are equivalent. It is supposed that arg z has its principal value and that the contour of integration is chosen such that the branch point $t = -z$ is excluded (Fig. 1). The integrand

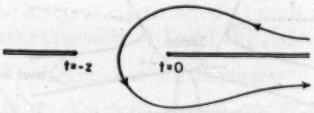

FIG. 1. The contour of integration of the Whittaker function in the t-plane with two branch points at $t = 0$ and $t = -z$ (refer to Eq. 1).

is rendered one-valued by taking $-\pi \leq \arg(-t) < \pi$ and taking that value of arg $\left(1 + \frac{t}{z}\right)$ which tends to zero as $t \to 0$ by a path lying inside the contour.

Our problem is to evaluate the asymptotic value of $W_{k,m}^{(n)}(z)$ for large values of n. For this purpose we shall use Debye's method of steepest descent.[6] It is found more convenient to perform the transformation

$$t = zt' \tag{3}$$

before applying the method. Dropping off the primes after completing the transformation we have

$$W_{k,m}^{(n)}(z) = -\frac{1}{2\pi i}\Gamma(k + \tfrac{1}{2} - m)e^{-z/2}z^{\frac{1}{2}+m} \times \int_\infty^{(0+)}(-t)^{-k-\frac{1}{2}+m}(1+t)^{k-\frac{1}{2}+m}e^{-zt}\,dt. \tag{4}$$

[6] Although Debye's method is well known in the literature of mathematics, a clear detailed presentation is contributed by Sommerfeld (3) and Jeffreys (4).

Consider the integral

$$I = \int_{\infty}^{(+0)} (-t)^{-k-\frac{1}{2}+m}(1+t)^{k-\frac{1}{2}+m}e^{-zt}\,dt \tag{5a}$$

$$= \int_{\infty}^{(+0)} F(t)\,dt \tag{5b}$$

$$= \int_{\infty}^{(+0)} \exp\left[f(t)\right] dt, \tag{5c}$$

where[7]

$$F(t) = (-t)^{-k-\frac{1}{2}+m}(1+t)^{k-\frac{1}{2}+m}e^{-zt} \tag{6a}$$

$$f(t) = (-k-\tfrac{1}{2}+m)\log(-t) + (k-\tfrac{1}{2}+m)\log(1+t) - zt. \tag{6b}$$

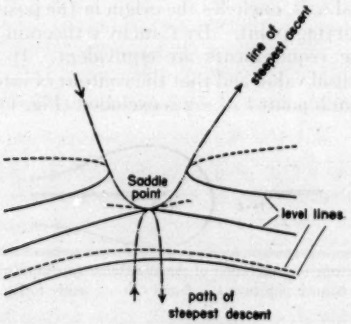

FIG. 2. The surface exp. $f(t)$ in the neighborhood of a saddle point.

If we can find the asymptotic expansion of I, then that of $W^{*}_{k,m}(z)$ can be calculated immediately because the asymptotic expansion of the Gamma function is well known. To calculate the asymptotic value of I, let us examine the variation of the integrand, Exp. $f(t)$. We observe that the magnitude of the integrand is greatest at the point where Re. $f(t)$ is greatest. Now if we plot the value of Re. $f(t)$ in the complex t-plane, and imagine its value as the elevation of a landscape, we have a mountainous region: Some parts of which constitute the valley where Re. $f(t)$ is small, while others consist of towering mountains where Re. $f(t)$ is large. Let us consider any two points A and B, not on a level line (that is, line along which Re. $f(t)$ = constant) and assume that Re. $f(t_A) >$ Re. $f(t_B)$, that is, A is at a higher elevation than B. Referring to Eq. 6b, we see that as $n \to \infty$, the value of the integrand Exp. $f(t)$ at A will be of exponentially higher order than that

[7] All the logarithms are restricted to the principal values.

at B so that the contribution to the integral in the neighborhood of B is negligible compared with that at A. Now from elementary calculus the Re. $f(t)$ attains an "extremum" when

$$\frac{\partial}{\partial x}[\text{Re. } f(t)] = \frac{\partial}{\partial y}[\text{Re. } f(t)] = 0.$$

$f(t)$ being an analytic function, this occurs at the point $\frac{df}{dt} = 0$. This extremum, like all extrema of the real and imaginary parts of an analytic function, is not a maximum or a minimum but a saddle point (Fig. 2). Now the contour of integration is chosen to pass through a saddle point and to lie elsewhere completely in the valley of the saddle; then as n increases indefinitely, the sides of valley become ever so steep that the main contribution to the value of the integral I may be attributed to that in the immediate neighborhood of the saddle point. Now the contribution to the value of the integral I near each saddle point can be readily evaluated by expanding $f(t)$ in a Taylor's series about the point; this will enable us to find the asymptotic value of the integral. Hence, we shall first find the locations of the saddle points, then the value of the integral contributed at each of these points, and finally the asymptotic expansion of I and $W^{(n)}_{k,m}(z)$.

LOCATIONS OF SADDLE POINTS[8]

The saddle points, as has been shown, are given by the condition $\frac{df}{dt} = 0$. Now

$$f'(t) = -\frac{z(t-t_1)(t-t_2)}{t(t+1)},$$

where t_1, t_2 are roots of the equation:

$$t^2 + \left(1 - \frac{2m-1}{z}\right)t + \frac{k-m+\frac{1}{2}}{z} = 0.$$

Hence $t = t_1, t_2$ are the two saddle points. More explicitly, they are given by

$$\left.\begin{matrix} t_1 \\ t_2 \end{matrix}\right\} = \frac{m}{z} - \frac{1}{2} - \frac{1}{2z} \pm \frac{1}{2}\sqrt{\left(1 - \frac{4k}{z} + \frac{4m^2}{z^2}\right) - \frac{4m-1}{z^2}}. \qquad (7)$$

For $\sigma = 0$ (sonic flow), $\Delta = \sqrt{\left(1 - \frac{4k}{z} + \frac{4m^2}{z^2}\right) - \frac{4m-1}{z^2}}$ approaches zero as $n \to \infty$ so that for large n and at sonic flow the two saddle

[8] The transformation in Eq. 3 keeps the saddle points fixed in the finite part of the t-plane.

points, for all practical purposes, coincide at the point $t_s = \frac{im_0}{z_0} - \frac{1}{2}$. For subsonic flow, $\Delta^2 > 0$; therefore, the two saddle points are images of each other on the line $t_x = -1/2$. For supersonic flow, $\Delta^2 < 0$; therefore, the two saddle points lie on the line $t_x = -1/2$ (see Fig. 3).

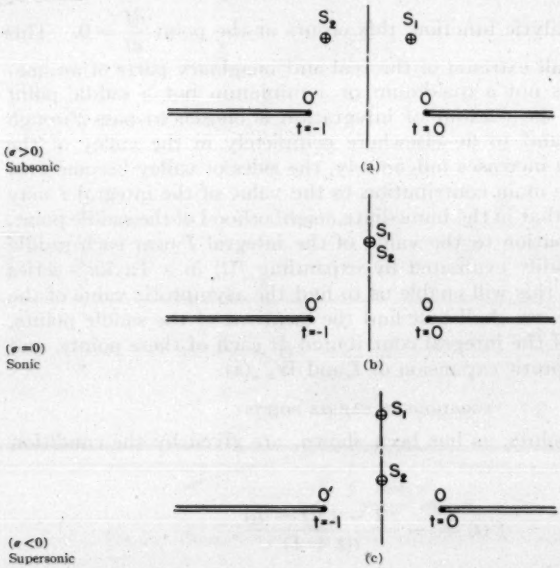

FIG. 3. Locations of the saddle points in the different flow regimes in the normalized t-plane.

EVALUATION OF INTEGRAL I NEAR THE SADDLE POINTS

Since the contribution to the value of the integral I (Eq. 5) is essentially concentrated at the neighborhood of the saddle points when n is large, we need only to evaluate the integral near the saddle points. In the neighborhood of these points (where $f'(t) = 0$), $f(t)$ can be expanded in a Taylor's series. Thus, near $t = t_1$

$$f(t) = f(t_1) - \frac{z}{2}\frac{t_1 - t_2}{t_1(t_1+1)}(t-t_1)^2 - \frac{z}{3}\frac{1}{t_1(t_1+1)}\left[1 - \frac{(2t_1+1)(t_1-t_2)}{t_1(t_1+1)}\right](t-t_1)^3 + \cdots. \quad (8a)$$

Similarly, near $t = t_2$

$$f(t) = f(t_2) - \frac{z}{2}\frac{t_2 - t_1}{t_2(t_2+1)}(t - t_2)^2 - \frac{z}{3}\frac{1}{t_2(t_2+1)}\left[1 - \frac{(2t_2+1)(t_2 - t_1)}{t_2(t_2+1)}\right](t - t_2)^3 + \cdots. \quad (8b)$$

The contribution to the value of the integral I near the saddle points $t = t_1$ and $t = t_2$, denoted by I_1 and I_2, respectively, are given by

$$I_1 = F(t_1)e^{i\gamma_1}\int_{-\epsilon}^{\epsilon} \exp\left[-\frac{z}{2}\frac{t_1 - t_2}{t_1(t_1+1)}s^2e^{2i\gamma_1} - \frac{z}{3}\frac{1}{t_1(t_1+1)}\left(1 - \frac{(2t_1+1)(t_1 - t_2)}{t_1(t_1+1)}\right)s^3e^{3i\gamma_1}\right]ds \quad (9a)$$

$$I_2 = F(t_2)e^{i\gamma_2}\int_{-\epsilon}^{\epsilon} \exp\left[-\frac{z}{2}\frac{t_2 - t_1}{t_2(t_2+1)}s^2e^{2i\gamma_2} - \frac{z}{3}\frac{1}{t_2(t_2+1)}\left(1 - \frac{(2t_2+1)(t_2 - t_1)}{t_2(t_2+1)}\right)s^3e^{3i\gamma_2}\right]ds. \quad (9b)$$

In these expressions the fourth and higher order terms in the Taylor's expansion are neglected and $(t - t_j)$ is replaced by $se^{i\gamma_j}$ $(j = 1, 2)$. The integration limits extend from $s = -\epsilon$ to $+\epsilon$, where ϵ is a small but finite number. The path of integration is chosen to be a small segment of a straight line so that the direction of the path is a constant. The angle γ will be chosen in the direction of the steepest descent, that is, the direction of maximum rate of change of elevation (that is, the real part of $f(t)$), because the contribution to the integral I outside the range $s = -\epsilon$ to $+\epsilon$ (which has been ignored in our calculations) will be least when γ is so chosen.

For low subsonic flow ($\Delta \to 1$) and high supersonic flow ($\Delta \to i\infty$), t_1 differs from t_2 considerably so that the second order term of Eq. 9 becomes quite large compared with the third order term. In fact, all higher order terms than the second have a negligible effect on the value of the integral provided n is large enough. So it is necessary only to consider the second order term to find the leading term in the asymptotic expansion.

At sonic speed, $t_1 = t_2$ and the second derivative in the Taylor's expansion vanishes. For this case, we need only consider the third order term.

For transonic flow, t_1 and t_2 are of the same order. It is then necessary to consider both the 2nd and 3rd order terms. It is slightly involved to investigate such cases with Debye's method. In Appendix B Langer's method is adopted for such transonic range.

1. Low Subsonic Flow ($\sigma > 1$)

Here the saddle points $t = t_1$, $t = t_2$ are far apart so that the third order term of Eq. 9 can be neglected completely. Hence

$$I_1 = F(t_1)e^{i\gamma_1} \int_{-\epsilon}^{\epsilon} \exp\left[-\frac{z}{2}\frac{t_1 - t_2}{t_1(t_1+1)} s^2 e^{2i\gamma_1}\right] ds \tag{10a}$$

$$I_2 = F(t_2)e^{i\gamma_2} \int_{-\epsilon}^{\epsilon} \exp\left[-\frac{z}{2}\frac{t_2 - t_1}{t_2(t_2+1)} s^2 e^{2i\gamma_2}\right] ds. \tag{10b}$$

Let

$$\frac{z}{2}\frac{t_1 - t_2}{t_1(t_1+1)} = \left|\frac{z}{2}\frac{t_1 - t_2}{t_1(t_1+1)}\right| e^{i\delta_1}. \tag{11}$$

Then

$$I_1 = F(t_1)e^{i\gamma_1} \int_{-\epsilon}^{\epsilon} \exp\left[-\left|\frac{z}{2}\frac{t_1 - t_2}{t_1(t_1+1)}\right| s^2 e^{2i\gamma_1 + i\delta_1}\right] ds. \tag{12}$$

As has been explained, for best approximation the angle γ_1 in Eq. 10 or 12 must be chosen in the direction of the steepest descent. This requires

$$\text{Im.}\,[e^{2i\gamma_1 + i\delta_1}] = 0, \quad \text{Re.}\,[e^{2i\gamma_1 + i\delta_1}] > 0. \tag{13}$$

The first requirement will guarantee that the path of integration is along a line perpendicular to the constant level lines in the neighborhood of the saddle point. It is, therefore, either in the direction of the line of steepest descent or the direction of the line of steepest ascent. The second requirement will guarantee that the path is in the direction of steepest descent because the integrand at any point along the path will not be greater than that at the saddle point (that is, the path lies in the valley). See Fig. 2.

It follows from Eq. 13 that

$$\gamma_1 = -\frac{\delta_1}{2}, \quad \pm\pi - \frac{\delta_1}{2}, \cdots. \tag{14}$$

Without loss of generality, let us choose $\gamma_1 = -\frac{\delta_1}{2}$. This is permissible because the sense of integration of the integral I_1 has not yet been specified. If we introduce the variable

$$\xi^2 = \left|\frac{z}{2}\frac{t_1 - t_2}{t_1(t_1+1)}\right| s^2 \tag{15}$$

$$I_1 = F(t_1)e^{i\gamma_1} \int_{-\epsilon\sqrt{\frac{z}{2}\frac{t_1-t_2}{t_1(t_1+1)}}}^{\epsilon\sqrt{\frac{z}{2}\frac{t_1-t_2}{t_1(t_1+1)}}} \exp\left[-\xi^2\right] d\xi \sqrt{\frac{2}{z}\frac{t_1(t_1+1)}{t_1 - t_2}}$$

$$= F(t_1)e^{i\gamma_1}\sqrt{\frac{2}{z}\frac{t_1(t_1+1)}{t_1 - t_2}} \int_{-\epsilon\sqrt{\frac{z}{2}\frac{t_1-t_2}{t_1(t_1+1)}}}^{\epsilon\sqrt{\frac{z}{2}\frac{t_1-t_2}{t_1(t_1+1)}}} e^{-\xi^2} d\xi.$$

For a fixed ϵ, and a sufficiently large n the integral is bounded from above by $\sqrt{\pi}$ (and, in fact, approaches the value $\sqrt{\pi}$ as n is increased indefinitely). Therefore, for a sufficiently large n

$$I_1 = F(t_1)e^{-\frac{i\delta_1}{2}}\sqrt{\left|\frac{2\pi}{z}\frac{t_1(t_1+1)}{t_1-t_2}\right|} \tag{16a}$$

that is

$$I_1 = F(t_1)\sqrt{\frac{2\pi}{z}\frac{t_1(t_1+1)}{t_1-t_2}}\,. \tag{16b}$$

Similarly,

$$I_2 = F(t_2)\sqrt{\frac{2\pi}{z}\frac{t_2(t_2+1)}{t_2-t_1}}\,. \tag{17}$$

We shall next proceed to relate the integral I with I_1 and/or I_2. For this purpose it is necessary to construct the constant level lines in the complex t-plane, especially those passing through the saddle points. First of all, we note that for low subsonic flow $\sigma > 1$ and $\Delta^2 > 0$ and that the saddle points (for $n \to \infty$)

$$\left.\begin{matrix} t_1 \\ t_2 \end{matrix}\right\} = \frac{im_0}{z_0} - \frac{1}{2} \pm \frac{\Delta}{2} \tag{18}$$

are image points with respect to the line $t_r = -1/2$. Referring to Fig. 4, let us denote the origin $t = 0$ by O and the branch point $t = -1$ by O', then

$$\overline{OS_1} = t_1$$
$$\overline{OS_2} = t_2$$
$$\overline{O'S_1} = t_1 + 1$$
$$\overline{O'S_2} = t_2 + 1.$$

Then by Eq. 11, since z is real, positive

$$\begin{aligned}\delta_1 &= \arg\left\{\frac{z}{2}\frac{t_1-t_2}{t_1(t_1+1)}\right\} \\ &= -\left[\arg(t_1) + \arg(t_1+1)\right] \\ &= -\left[\arg\overline{OS_1} + \arg\overline{O'S_1}\right].\end{aligned}$$

If we now construct the internal bisector S_1T_1 of the angle OS_1O' and denote its argument by α, then it follows from elementary geometry that

$$\arg\overline{OS_1} + \arg\overline{O'S_1} = 2\alpha$$

so that

$$\delta_1 = -2\alpha. \tag{19a}$$

By Eq. 14, we have

$$\gamma_1 = \alpha. \tag{19b}$$

That is, the direction of the steepest descent at S_1 is in the direction of the internal bisector of OS_1O'. Similarly, one can show that the direction of the steepest descent at S_2 is in the direction of the external bisector of OS_2O'. These lines are shown in Fig. 4 and are indicated by the arrows.

The lines of constant elevation (that is, the level lines) passing through the saddle point S_1 ($t = t_1$) are given by

$$\text{Re.}\,[f(t)] = \text{Re.}\,[f(t_1)]. \tag{20}$$

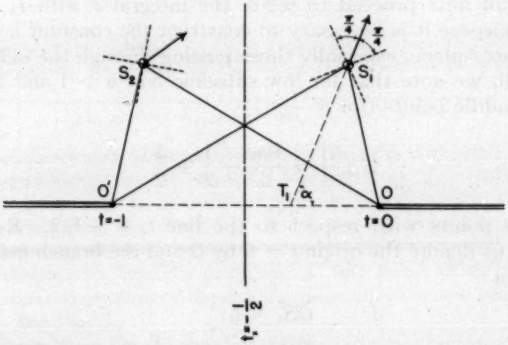

FIG. 4. Directions of steepest descent (indicated by the arrows) at the two saddle points, S_1 and S_2, for the subsonic case ($\sigma > 0$). Dotted lines through the saddle points show constant level lines.

In the immediate neighborhood of S_1 along its level lines, it follows from Eqs. 8*a* and 20 that

$$\text{Re.}\left[-\frac{z}{2}\frac{t_1 - t_2}{t_1(t_1+1)}\,s^2 e^{2i\bar{\gamma}_1}\right] = 0,$$

where $\bar{\gamma}$ = argument of either level line through S_1. Introducing δ_1 (Eq. 11) the above expression is equivalent to

$$\cos(\pi + 2\bar{\gamma}_1 + \delta_1) = 0$$

or

$$\bar{\gamma}_1 = \gamma_1 \pm \frac{\pi}{4}. \tag{21}$$

In other words, the constant level lines through the saddle point S_1 are inclined at a 45-deg. angle to the line of steepest descent. A similar conclusion holds for the saddle point S_2. These lines are shown dotted in Fig. 4. It is seen at once that the constant level lines through the saddle point divide the neighborhood of the point into consecutive regions of low and high lands (that is, valleys and mountains). Furthermore, it is also clear from the figure that the saddle point S_2 is at a higher elevation than S_1.

At a distance from a saddle point the Taylor's expansion, Eq. 8, converges so slowly that it is no longer valid to use only the second order term in $f(t)$. Then Eq. 20 has to be used to calculate the level lines. Remembering that as $n \to \infty$, m becomes purely imaginary while k and z remain real, Eq. 6b becomes, for large value of n,

$$\frac{1}{z}\,\text{Re.}\,[f(t)] = \frac{k_0}{z}\log\left|\frac{t}{1+t}\right| - \frac{m_0}{z_0}\left[\arg(-t) + \arg(1+t)\right] - \text{Re.}\,(t).$$

Introducing the real and imaginary part of t, namely, $t = t_x + it_y$, the above equation becomes

$$\frac{1}{z}\,\text{Re.}\,[f(t)] = \frac{1}{2}\frac{k_0}{z_0}\log\left[\frac{t_x^2 + t_y^2}{(t_x+1)^2 + t_y^2}\right] - \frac{m_0}{z_0}\left[\tan^{-1}\left(\frac{-t_y}{-t_x}\right) + \tan^{-1}\left(\frac{t_y}{t_x+1}\right)\right] - x.$$

If we define as "elevation" the product of Re. $f(t)$ and $\frac{1}{z}$, the latter being constant in the t-plane, the "elevation" of the line $t_x = -1/2$ is constant and equals

$$\frac{1}{z}\,\text{Re.}\,[f(-\tfrac{1}{2})] = \tfrac{1}{2}. \qquad (22b)$$

If we define an "elevation" function E, and measure this function with respect to the datum line $t_x = -1/2$

$$E = \frac{1}{z}\left[\text{Re.}\,f(t) - \text{Re.}\,f(-\tfrac{1}{2})\right] \qquad (22c)$$

$$= \frac{1}{2}\frac{k_0}{z_0}\log\left[\frac{t_x^2 + t_y^2}{(t_x+1)^2 + t_y^2}\right] - \frac{m_0}{z_0}\left[\tan^{-1}\left(\frac{-t_y}{-t_x}\right) + \tan^{-1}\left(\frac{t_y}{t_x+1}\right)\right] - (t_x + \tfrac{1}{2}).$$

By a change of variable $t_x = t_{x'} - 1/2$, it is seen that the lines of constant "elevation" are skew-symmetric with respect to the line $t_x = -1/2$,

so that whenever we have a mountain on the left we have an exactly similar valley on the right and vice versa.

We note first of all that $E = -\infty$ at $t = 0$, and that $E = +\infty$ at $t = -1$. Also from Eq. 22c it is seen that for any given y, E approaches negative infinity as t_x approaches positive infinity, and E approaches positive infinity as t_x approaches negative infinity. Consequently, the high land to the right of the path of steepest descent at S_1 (Fig. 4) and the low land to the left of the path of steepest descent at S_2 extend only to some finite region. This is explicitly shown in Fig. 5 in which contour map for Re. $f(t)$ for $n = 16$ and $\sigma = 2.42$ or $M = 0.68$ is drawn. The fact that there is a "dead valley" region to the left of S_2, bounded by the cut from $t = -1$ to $-\infty$ and the constant level lines through S_2 should not be overlooked. In Fig. 5, the contour map is not symmetrical with respect to the line $t_x = -1/2$ because n is finite ($=16$). The contour map for $n = \infty$ is shown in Fig. 6. It is seen that Figs. 5 and 6

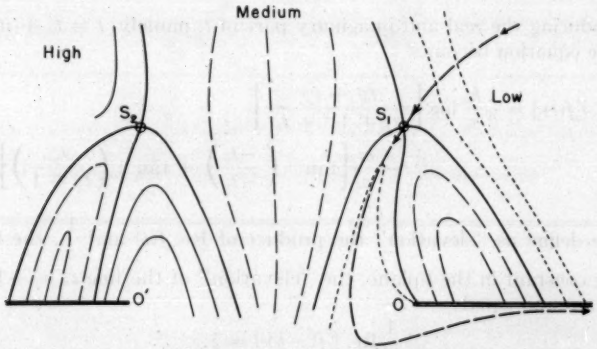

FIG. 5. Contour map of Re. $f(t)$ for $n = 16$, $\sigma = 2.42$ ($M = 0.68$). The required contour of integration is shown by the heavy dashed line.

are essentially similar. We are now in a position to relate I with I_1 and/or I_2.

We recall that the path of integration for integral I should start from $+\infty$ on the real axis, encircle the origin $t = 0$ in the positive sense, exclude the branch point $t = -1$ and return to $+\infty$ of the real axis. The best path of integration for I will pass through the saddle point S_1 only and lie elsewhere entirely in its valley. Then the main contribution to the integral I, when $n \to \infty$ comes from the saddle point S_1. In Fig. 5 the path of integration is shown by the heavy broken line. From the figure it is also seen that the sense of integration of the integral I is opposite to that of I_1 (compare Figs. 4 and 5) so that as $n \to \infty$ we have

$$I = -I_1 = -F(t_1)\sqrt{\frac{2\pi}{z}\frac{t_1(t_1+1)}{t_1 - t_2}}. \tag{23}$$

Now the asymptotic expansion of the Gamma function $\Gamma(k + 1/2 - m)$ is

$$\Gamma(k + \tfrac{1}{2} - m) = \sqrt{2\pi}(k - \tfrac{1}{2} - m)^{k-m} e^{-(k-\frac{1}{2}-m)}. \tag{24}$$

Substituting Eqs. 23 and 24 in Eq. 4, we obtain the asymptotic expansion of $W^{(n)}_{k,m}(z)$ for large value of n and $\sigma > 1$

$$W^{(n)}_{k,m}(z) \sim -iF(t_1) z^m (k - \tfrac{1}{2} - m)^{k-m} e^{-\left(k+\frac{s-1}{2}-m\right)} \sqrt{\frac{t_1(t_1+1)}{t_1 - t_2}}. \tag{25}$$

In actual calculation, however, Eq. 25 is not too convenient for numerical evaluation because the various quantities involved at the right hand side of the above equation are complex although we know that $W^{(n)}_{k,m}(z)$ must be real as is evident from the differential equation satisfied by the function. Moreover, the saddle-point t_1 in Eq. 25 has to be calcu-

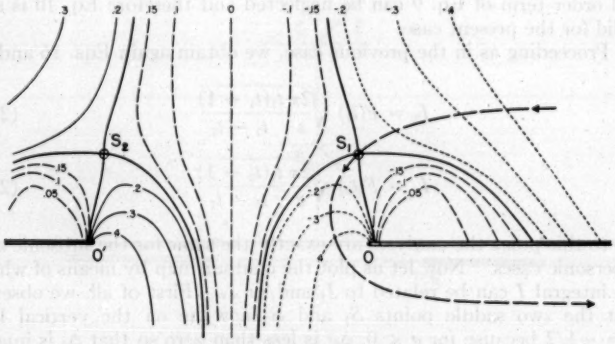

FIG. 6. Same as Fig. 5 except $n = \infty$.

lated from Eq. 7 which involved a lot of higher order terms. The same is true for the asymptotic expansion of the Gamma function. In order to reduce Eq. 25 to a real form and to maintain a consistent order of magnitude, we shall expand into descending power series of n the various quantities involved in the equation. This is done in Appendix A and the result is

$$W^{(n)}_{k,m}(z) \sim -\frac{n^k z_0{}^k}{\sqrt{\Delta_0}} \left[\frac{\frac{1}{2} + \frac{k_0}{z_0} + \frac{\Delta_0}{2} \left(\frac{k_0{}^2}{z_0{}^2} + \frac{m_0{}^2}{z_0{}^2} \right)}{\frac{1}{2} + \frac{k_0}{z_0} - \frac{\Delta_0}{2}} \right]^{\frac{k}{2}}$$

$$\times e^{\left\{ nk_0 - n\frac{z_0\Delta_0}{2} - nm_0 \tan^{-1} \left[\frac{\frac{m_0}{z_0}\frac{k_0}{z_0}(\Delta_0 - 1) + \frac{2m_0{}^3}{z_0{}^3}}{\frac{k_0{}^2}{z_0{}^2} + \frac{m_0{}^2}{z_0{}^2}\Delta_0 - 2\frac{k_0}{z_0}\frac{m_0{}^2}{z_0{}^2}} \right] \right\}} \left[1 + O\left(\frac{\log n}{n} \right) \right] \tag{26}$$

where

$$k = -nk_0 + k_1$$

$$\Delta_0 = \left[1 + \frac{4k_0}{z_0} - \frac{4m_0^2}{z_0^2}\right]^{\frac{1}{2}}.$$

It should be mentioned that Eq. 25 is even valid for $m = inm_0 + m_1 + O\left(\frac{1}{n}\right)$. However, for such m, the asymptotic expansion cannot be reduced to a real form (see Appendix A). Since the asymptotic expansion for $\sigma < 0$ the supersonic case, is very similar to that given by Eq. 25 we shall treat this case next instead of the case $\sigma = 0$.

2. Supersonic Flow ($\sigma < 0$, $|\Delta| \to \infty$)

Here again as $n \to \infty$ the two saddle points are far apart so that the 3rd order term of Eq. 9 can be neglected and therefore Eq. 10 is still valid for the present case.

Proceeding as in the previous case, we obtain again Eqs. 16 and 17

$$I_1 = F(t_1)\sqrt{\frac{2\pi}{z}\frac{t_1(t_1+1)}{t_1 - t_2}} \tag{27a}$$

$$I_2 = F(t_2)\sqrt{\frac{2\pi}{z}\frac{t_2(t_2+1)}{t_2 - t_1}}. \tag{27b}$$

Up to this point the analyses are exactly the same for the subsonic and supersonic cases. Now let us plot the contour map by means of which the integral I can be related to I_1 and/or I_2. First of all, we observe that the two saddle points S_1 and S_2 now lie on the vertical line $t_x = -1/2$ because for $\sigma < 0$, Δ_0^2 is less than zero so that Δ_0 is imaginary. Let $\Delta_0 = i\Delta_1$. The saddle points ($n \to \infty$) are given by Eq. 18

$$\left.\begin{matrix} t_1 \\ t_2 \end{matrix}\right\} = -\tfrac{1}{2} + i\left(\frac{m_0}{z_0} \pm \frac{\Delta_1}{2}\right). \tag{28}$$

These points are shown in Fig. 7. Remembering that $\arg(t_1 - t_2)$ is $\frac{\pi}{2}$ now instead of 0, one can show (in exactly the same manner as before) that the direction of steepest descent at the saddle points S_1, S_2 are, respectively,

$$\gamma_1 = \frac{\pi}{4} \tag{29a}$$

$$\gamma_2 = -\frac{\pi}{4}. \tag{29b}$$

These directions are also shown in Fig. 7 where they are indicated by lines with arrows. The "elevation" of Re. $f(t)$ in the complex t-plane, for $n \to \infty$, is still given by Eq. 22a, namely

$$\frac{1}{z}\,\mathrm{Re.}\, f(t) = \frac{1}{2}\frac{k_0}{z_0}\log\left[\frac{t_x^2 + t_y^2}{(t_x+1)^2 + t_y^2}\right] - \frac{m_0}{z_0}\left[\tan^{-1}\left(\frac{-t_y}{-t_x}\right) + \tan^{-1}\left(\frac{t_y}{t_x+1}\right)\right] - t_x. \quad (30a)$$

Again $t_x = -1/2$ is a level line (line along which Re. $f(t)$ is constant) so that both saddle points S_1 and S_2 are now at the same elevation.

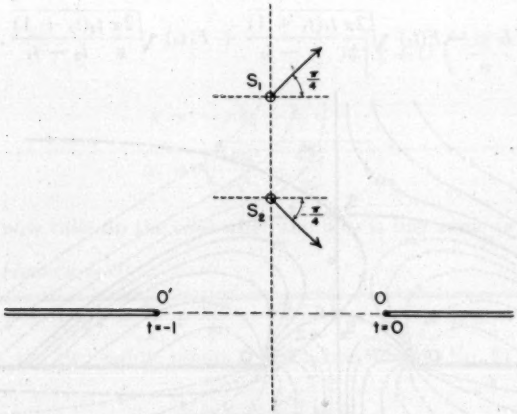

FIG. 7. Directions of steepest descent (indicated by the arrows) at the two saddle points, S_1 and S_2, for the supersonic case ($\sigma < 0$).

Also, if we measure all elevation change with respect to the line $t_x = -1/2$ (or the saddle points) we have

$$E = \frac{1}{2}\frac{k_0}{z_0}\log\left[\frac{t_x^2 + t_y^2}{(t_x+1)^2 + t_y^2}\right] - \frac{m_0}{z_0}\left[\tan^{-1}\left(\frac{-t_y}{-t_x}\right) + \tan^{-1}\left(\frac{t_y}{t_x+1}\right)\right] - (t_x + \tfrac{1}{2}). \quad (30b)$$

The contour map for $n = \infty$ and $\sigma = -0.04$ or $M = 1.08$ is shown in Fig. 8. The contour lines are therefore skew-symmetric with respect to the line $t_x = -1/2$ so that the valleys at one side of it correspond

to mountains at the other side and vice versa. Also, it is seen that for any t_y, $E \to -\infty$ as $t_x \to +\infty$ so that the mountainous region to the right of the datum line $t_x = -1/2$ is confined to a finite domain. We shall now proceed to relate I with I_1 and/or I_2. Since the path of integration for the integral I should start from $+\infty$ on the real axis, encircle the origin $t = 0$ in the positive sense, but exclude the branch point $t = -1$, and return to $+\infty$ of the real axis, and since the sense of the integrals I_1, I_2 are, respectively, in the direction of $\pm \frac{\pi}{4}$ (Fig. 6); therefore the integral I is related to I_1 and I_2 by

$$I = -I_1 + I_2$$

or

$$I = -F(t_1)\sqrt{\frac{2\pi}{z}\frac{t_1(t_1+1)}{t_1-t_2}} + F(t_2)\sqrt{\frac{2\pi}{z}\frac{t_2(t_2+1)}{t_2-t_1}}. \tag{31}$$

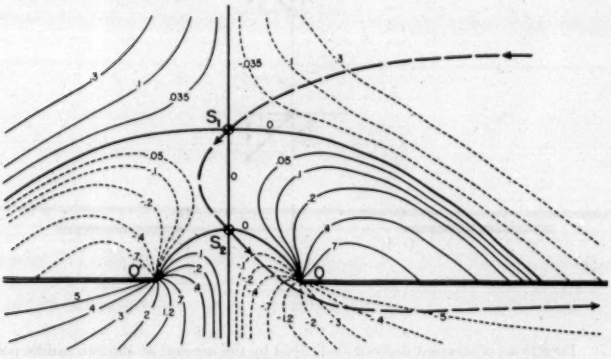

FIG. 8. Contour map of Re. $f(t)$ for $n = \infty$, $\sigma = 0.04$ ($M = 1.08$).

Now the asymptotic expansion of the Gamma function is given by Eq. 24; therefore the asymptotic expansion of $W^{(n)}_{k,m}(z)$ for large value of n and $|\Delta| \gg 0$ is

$$W^{(n)}_{k,m}(z) \sim iz^m(k-\tfrac{1}{2}-m)^{k-m}e^{-\left(k+\frac{s-1}{2}-m\right)} \times\left[-F(t_1)\sqrt{\frac{t_1(t_1+1)}{t_1-t_2}} + F(t_2)\sqrt{\frac{t_2(t_2+1)}{t_2-t_1}}\right]. \tag{32}$$

The above equation is still true for $m = inm_0 + m_1 + O(1/n)$. For $m_1 = 0$, it should be possible to reduce the above expression to a real

form. This is in fact the case and the result is

$$W^{(n)}_{k,m}(z) \sim -\frac{2n^k z_0^k}{\Delta_1}\left[\frac{k_0^2}{z_0^2}+\frac{m_0^2}{z_0^2}\right]^{\frac{k}{2}} e^{nk_0 - nm_0 \tan^{-1}\left(\frac{m_0}{-k_0}\right)}$$

$$\times \cos\left[nm_0 \log\left(\frac{-\frac{k_0}{z_0}+\frac{2m_0^2}{z_0^2}+\frac{m_0\Delta_1}{z_0}}{\sqrt{\frac{k_0^2}{z_0^2}+\frac{m_0^2}{z_0^2}}}\right)-\frac{nz_0\Delta_1}{2}\right.$$

$$\left.-(k+\tfrac{1}{4})\pi+2k\tan^{-1}2\left(\frac{m_0}{z_0}+\frac{\Delta_1}{2}\right)+k\tan^{-1}\frac{m_0}{z_0}\right]$$

$$\times\left[1+O\left(\frac{\log n}{n}\right)\right], \quad (33)$$

where

$$k = -nk_0 + k_1$$

$$\Delta_1 = \sqrt{\frac{4m_0^2}{z_0^2}-\frac{4k_0}{z_0}-1}\,.$$

We shall now take up the case when the flow is just sonic ($\sigma = 0$).

3. Sonic Flow ($\sigma = 0$)

When $\sigma = 0$, $\sqrt{1-\frac{4k}{z}+\frac{4m^2}{z^2}-\frac{4m-1}{z}} \to \Delta_0 = 0$ as $n \to \infty$. Consequently, the two saddle points t_1 and t_2 coincide at (Eq. 8)

$$t_1 = t_2 = \frac{im_0}{z_0}-\frac{1}{2} \quad (34)$$

as n approaches infinity.

This point lies on the line of symmetry $t_x = -1/2$ (see Fig. 9). Since $t_1 - t_2 = 0$, Eq. 9 becomes simply

$$I_1 = F(t_1)e^{i\gamma_1}\int_{-\epsilon}^{\epsilon}\exp\left[-\frac{z}{3}\frac{1}{t_1(t_1+1)}s^3e^{3i\gamma_1}\right]ds, \quad (35)$$

where γ_1 is rendered single valued by $|\gamma_1| \leq \frac{\pi}{2}$.

The direction of steepest descent or ascent is in the direction of the maximum rate of change of elevation and is therefore given by:

$$\text{Im.}\left[-\frac{z}{3}\frac{1}{t_1(t_1+1)}s^3e^{3i\gamma}\right] = 0 \quad (36)$$

for reasons explained already in connection with Eq. 13. Referring to Fig. 9, we observe that

$$\arg(t_1) + \arg(t_1 + 1) = \pi.$$

Hence Eq. 36 can be written simply as

$$\sin(3\gamma - \pi) = 0$$

or

$$\gamma = 0, \quad \pm\frac{\pi}{3}. \tag{37}$$

Consider the following integrals

$$I_{11} = F(t_1)\int_0^{\epsilon} \exp\left[-\frac{z}{3}\frac{1}{t_1(t_1+1)}s^3\right]ds \tag{38a}$$

$$I_{12} = F(t_1)\int_{-\epsilon}^{0} \exp\left[-\frac{z}{3}\frac{1}{t_1(t_1+1)}s^3\right]ds \tag{38b}$$

$$I_{13} = F(t_1)e^{i\frac{\pi}{3}}\int_0^{\epsilon} \exp\left[-\frac{z}{3}\frac{1}{t_1(t_1+1)}s^3e^{i\pi}\right]ds \tag{38c}$$

$$I_{14} = F(t_1)e^{i\frac{\pi}{3}}\int_{-\epsilon}^{0} \exp\left[-\frac{z}{3}\frac{1}{t_1(t_1+1)}s^3e^{i\pi}\right]ds \tag{38d}$$

$$I_{15} = F(t_1)e^{-i\frac{\pi}{3}}\int_0^{\epsilon} \exp\left[-\frac{z}{3}\frac{1}{t_1(t_1+1)}s^3e^{-i\pi}\right]ds \tag{38e}$$

$$I_{16} = F(t_1)e^{-i\frac{\pi}{3}}\int_{-\epsilon}^{0} \exp\left[-\frac{z}{3}\frac{1}{t_1(t_1+1)}s^3e^{-i\pi}\right]ds. \tag{38f}$$

Introducing the variable $\eta^3 = \frac{z}{3}\left|\frac{1}{t_1(t_1+1)}\right|s^3$ instead of s, integrating with respect to η and then letting n become very large, it is easy to show that the integrals, I_{11}, I_{14}, I_{16}, are not convergent while I_{12}, I_{13}, I_{15} become

$$I_{12} = F(t_1)\Gamma(\tfrac{1}{3})\sqrt[3]{\frac{|t_1(t_1+1)|}{9z}} \tag{39a}$$

$$I_{13} = F(t_1)\Gamma(\tfrac{1}{3})e^{i\frac{\pi}{3}}\sqrt[3]{\frac{|t_1(t_1+1)|}{9z}} \tag{39b}$$

$$I_{15} = F(t_1)\Gamma(\tfrac{1}{3})e^{-i\frac{\pi}{3}}\sqrt[3]{\frac{|t_1(t_1+1)|}{9z}}. \tag{39c}$$

The path of integration of the six integrals of Eq. 38 is shown in Fig. 9. Since I_{11}, I_{14}, I_{16} are divergent, their paths must be on the mountain sides of the saddle and in the direction of steepest ascent, while the path of integration for I_{12}, I_{13}, I_{15} are in the direction of steepest descent. These are shown in Fig. 9 by heavy lines with arrows. The level lines passing through the saddle point are given by

$$\text{Re.}\,[f(t)] = \text{Re.}\,[f(t_1)]. \tag{40a}$$

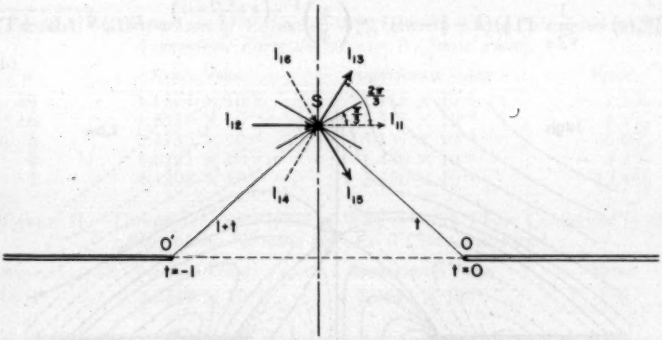

FIG. 9. Paths of integration through the saddle point S for the sonic case ($\sigma = 0$). ($M = 1$.)

In the neighborhood of the saddle point, the above equation reduces to

$$\text{Re.}\left[-\frac{z}{3}\frac{1}{t_1(t_1+1)}s^3e^{3i\bar{\gamma}}\right] = 0.$$

That is,

$$\cos(3\bar{\gamma} - \pi) = 0$$

$$\bar{\gamma} = \pm\frac{\pi}{6}, \quad \frac{\pi}{2}. \tag{40b}$$

These directions are shown in Fig. 9. The constant level lines through the saddle point, therefore, bisect the angles between the lines of steepest ascent and descent.

Using Eq. 40a, or 22c, we can now sketch the contour map in the complex t-plane. This is shown in Fig. 10. Remembering the path of the integral I should start at $+\infty$ on the real axis, encircle the origin $t = 0$ in the positive direction, exclude the point $t = -1$ and return to $+\infty$ on the real axis, we see at once (compare Figs. 9 and 10).

$$I = -I_{13} + I_{15}$$

$$= -i\sqrt{3}\Gamma(\tfrac{1}{3})F(t_1)\sqrt[3]{\frac{|t_1(t_1+1)|}{9z}}. \tag{41}$$

Hence, by Eq. 4,

$$W^{(n)}_{k,m}(z) \sim \frac{1}{2\pi}\,\Gamma(\tfrac{1}{3})\Gamma(k+\tfrac{1}{2}-m)\left(\frac{z}{3}\right)^{\frac{1}{3}} e^{-\frac{z}{2}} z^m F(t_1)\sqrt[3]{|t_1(t_1+1)|}. \quad (42a)$$

Replacing the Gamma function by its asymptotic expansion (Eq. 24), we have

$$W^{(n)}_{k,m}(z) \sim \frac{1}{\sqrt{2\pi}}\,\Gamma(\tfrac{1}{3})(k-\tfrac{1}{2}-m)^{k-m}\left(\frac{z}{3}\right)^{\frac{1}{3}} e^{-\left(k+\frac{z-1}{2}-m\right)} z^m F(t_1)\sqrt[3]{|t_1(t_1+1)|}. \quad (42b)$$

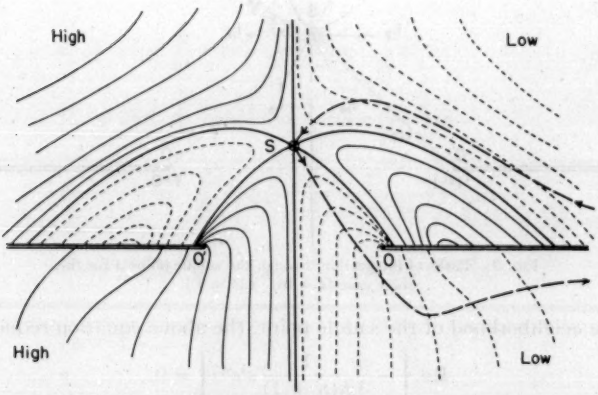

FIG. 10. Contour map of Re. $f(t)$ in the neighborhood of the saddle point S. According to Debye's theory the required contour of integration (the heavy dashed line) is replaced by two short line elements turning sharply at S.

The above equation is even valid for $m = inm_0 + m_1 + O(1/n)$. When $m_1 = 0$, the right hand side of the above equation should be reducible to a real form which is more convenient for numerical calculation. The result is

$$W^{(n)}_{k,m}(z) \sim \frac{n^{k+\frac{1}{2}}}{\sqrt{2\pi}}\,\Gamma(\tfrac{1}{3})\left(\frac{z_0}{3}\right)^{\frac{1}{3}} z_0{}^k e^{\left\{nk_0-2nm_0\tan^{-1}\frac{2m_0}{z_0}\right\}}\left(\frac{m_0{}^2}{z_0{}^2}+\frac{1}{4}\right)^{k-\frac{1}{2}}\left[1+O\left(\frac{1}{n}\right)\right], \quad (43)$$

where

$$k = -nk_0 + k_1.$$

The ratio of $W^{(n)}_{nk_0+\bar{k},m}(z)$ to $W^{(n)}_{nk_0,m}(z)$ as $n \to \infty$ can be calculated by

putting, respectively, $k_1 = \bar{k}$ and 0 in Eq. 43. Thus, we have simply

$$\frac{W^{(n)}_{n\bar{k}_0+\bar{k},m}(z)}{W^{(n)}_{n\bar{k}_0,m}(z)} \sim n^{\bar{k}} z_0^{\bar{k}} \left(\frac{1}{4} + \frac{m_0^2}{z_0^2}\right)^{\bar{k}}.$$

We shall now examine the accuracy of our asymptotic formulas. There is little reason to believe the asymptotic expansion holds for moderate values of n yet Table I shows that even for $n = 2$, the error (12 per cent)

TABLE I.—*Comparison of Values of* $W^{(n)}_{k,m}(z)$ *with Those Calculated by the Asymptotic Formula for* $\sigma = 0$ *(Sonic Flow).*

n	Exact Value	Approximate Value	Error
20	1.1424×10^{-5}	1.115×10^{-5}	− 1.5%
16	1.4426×10^{-4}	1.378×10^{-4}	− 4.5%
6	5.4735×10^{-2}	5.137×10^{-2}	− 6.0%
4	1.6221×10^{-1}	1.489×10^{-1}	− 8.2%
2	4.4298×10^{-1}	3.880×10^{-1}	−12.4%

TABLE II.—*Comparison of Values of* $W^{(n)}_{k,m}(z)$ *with Those Calculated by the Asymptotic Formula for* $\sigma > 0$ *(Subsonic Flow).*

n	Exact Value	Approximate Value	Error
16	6.2344×10^{-6}	6.2643×10^{-6}	1%

is not too great. The exact values were computed from the well-known series expansion of the Whittaker function (2). For the accuracy of figures indicated, up to approximately 32 terms of the series are needed. These involve complex gamma functions for which no tables are yet available.

COMPARISON OF DEBYE'S AND LANGER'S METHODS

From the above analysis and that given in Appendix B, it is seen that both Debye's method of steepest descent and Langer's method can be applied to find the asymptotic solution of the Whittaker function $W^{(n)}_{k,m}(z)$ as the parameter n increases indefinitely. A comparison of the two methods from the experience of the present investigation may be useful for future study.

(*a*) Debye's method is essentially a method of finding the asymptotic expansion of a function defined by a complex integral. It is therefore applicable to any particular solution of the differential equation when the latter is known in the integral form. On the other hand the Langer's method is applied directly to the differential equation with a transition point (the point is $\sigma = 0$, in our case). A general expression for the asymptotic solution which covers all the range of the independent variable of the differential equation can be obtained in which two arbitrary functions of the parameter n are involved. For any particular solution of the differential equation, it is only necessary to identify

properly the two arbitrary functions from "boundary conditions" defining the particular solution.

(*b*) Debye's method of steepest descent is, in general, longer than the Langer's method. However, as mentioned already, it is a method designed principally for finding the asymptotic expansion of functions defined by a complex integral.

(*c*) In the immediate neighborhood of the transion point, Debye's method is more involved. Langer's method is particularly favorable for such a range owing to the favorable behavior of Bessel function used.

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(*To be concluded.*)

NOTES FROM THE NATIONAL BUREAU OF STANDARDS*

THE KINORAMA—A DEVICE FOR EVALUATING AIRPORT APPROACH LIGHTS

Fog at airports constitutes a serious hazard to air traffic. Although extensive flight tests have been made to determine which of many proposed approach-light systems is best suited for landing in fog, these tests have thus far failed to bring about agreement on any one system. To supply this need, the National Bureau of Standards, in cooperation with the Navy Bureau of Aeronautics, has developed a device known as the kinorama with which a pilot can evaluate the guidance provided by a system of approach lights without leaving the ground. Perhaps more important in the long range than determining the best approach-light system will be the use of the kinorama in providing pilots with the vital training for landing in fog which can now be acquired only through hazardous experience.

The use of the kinorama makes it possible for the pilot to see an approach-light configuration speeding by him just as he would in flying over it in fog. The course followed in such a simulated flight is determined by the pilot himself, who acts in response to the lights that are visible to him. By recording the paths thus followed, it is possible to obtain objective records reflecting the merits of the light system. While the device is not a substitute for flight testing for the final approval of a system, it is inexpensive, objective, free from hazards, and not dependent upon the occurrence of fog.[1]

In spite of the excellent electronic equipment which has been developed for guiding pilots in fog, safe landings in bad visibility are not yet a common practice because there remains the problem of making the final contact with the runway. Where so much is at stake, it is desirable, if possible, to make this final contact visually rather than to trust the operation of electronic devices. It is the purpose of the approach lights to furnish an earlier visual contact with the ground.

Many engineers and pilots have attempted to design systems of lights to guide pilots in the critical seventeen seconds between the time the aircraft is near enough to the ground to be within the reach of lights and the pilot's first sight of the runway lighting. Practically all discussion of the merits of these different systems has revolved about

* Communicated by the Director.

[1] For more complete details see "The Kinorama as an Evaluator in Aviation Lighting," by F. C. BRECKENRIDGE, Preprint No. 21 of the National Technical Conference of the Illuminating Engineering Society, Sept. 8–12, 1952. Abstract published in *Illuminating Engineering*, September 1952.

stationary aspects of their designs, such as the interpretation of geometrical figures, the visual range of the lights, and the possibility of seeing lights in different locations. It is quite evident, however, that approach lights cannot be seen in stationary arrangements by pilots except in the unusual cases of fixed balloons and helicopters.

Two devices which incorporate the natural apparent motions of landing aircraft have been developed for the study of approach-light systems. One of these, called the cyclorama,[2] has been used for some time for demonstrations in England. This device, however, is limited to configurations in a single plane and hence cannot be used for evaluating a three-dimensional arrangement of lights such as the slope-line system. The second device, the NBS kinorama, undertakes a more complete simulation than the cyclorama and includes the three-dimensional aspects of approach-light systems.

Two models of the kinorama have been constructed at the National Bureau of Standards under the direction of F. C. Breckenridge. After the original experimental or laboratory model had demonstrated that it is possible to achieve the desired flight simulations, an improved prototype kinorama was constructed.

The basic principle of the kinorama is to represent an approach lighting system by a small, movable model whose motions are the reciprocal of those which a plane would have in landing over the lighting system. Because the pilot's reaction is the true test of any system being evaluated, the motions of the model must be those which would normally appear to follow the pilot's use of his controls. In the kinorama, the approach lights are represented by fluorescent miniatures of lighting units mounted on an endless tape which moves continuously toward the observer, simulating the plane's forward motion over the approach and runway lights. For lighting systems comprised of units that are all in one plane, it is only necessary to represent the lights by small spots of fluorescent material on the surface of the tape. However, for systems like the slope-line system in which the third dimension is significant the lighting units are represented by miniatures which include the third dimension.

To simulate the lighting system on a sufficiently small scale so that over-all dimensions are kept within practical limits, the point of observation must be placed very near to the tape in order to make the view correspond to that from points along the approach path. This is accomplished by the use of a telescope with negligible magnification and an objective approximately 0.5 inch in diameter. With a larger objective, it would not be feasible to bring the point of observation sufficiently near the lighting units and only the first stages of an approach could be simulated. With a telescope of sufficient length

[2] J. W. SPARKE AND H. F. RINGE, "A Laboratory Device for Simulating Visual Landings in Bad Weather," Roy. Air Est. Report El. 1460.

the pilot can sit in his seat normally and see the lights as if he were coming down among them. The telescope also simplified the problem of simulated banking. It was only necessary to install a dove prism at a suitable location in the telescope and rotate this prism to obtain a realistic simulation of a lighting system as seen from a banking plane.

The laboratory model is built on the fuselage of an airplane with its associated instruments and controls. Hinged plywood platforms, which move relative to one another, provide the apparent motions of flight. In the prototype kinorama, the cockpit, instruments, and controls of a Link trainer take the place of those of the airplane and the plywood platforms are replaced by rectangular frames of aluminum alloy. A true vertical motion has been substituted for the hinged movement. All the angular motions are simulated optically which simplified the mechanical design and made it possible to provide the desired yaw movements with less width. Servomotors control all movements of the kinorama including a recorder which provides a permanent record of each "landing." Special equipment allows automatic presetting of any one of nine initial landing states—that is, the selection of one of nine combinations of three altitudes with three transverse positions, combined with a predetermined yaw, bank and pitch for each state. This flexibility of presetting is sufficient to meet the needs of a wide variety of landing problems.

Evaluation of three different types of visual aids has been undertaken with the kinorama: (1) a comparison of three "over-run"[3] lighting systems designed at Wright-Patterson Air Force Base, utilizing different types of lighting units; (2) a comparison of the three approach-light systems proposed for standardization by the International Civil Aviation Organization; and (3) a study of longitudinal and transverse markings for runways. The results of NBS tests with the "over-run" systems and the runway markings have shown satisfactory agreement with corresponding flight tests. The investigation of the approach light systems are as yet incomplete; preliminary results obtained with the laboratory kinorama will be supplemented by tests with the prototype model.

In addition to its usefulness for evaluating lighting systems the kinorama gives promise of being a valuable training device. The use of approach-light systems in good weather does not make a pilot proficient in using them in fog. The irregularity and suddeness with which fogs occur, on the other hand, make it impracticable for a pilot to obtain his training by gradually flying in more and more difficult weather. The kinorama can be used to supplement experience in landings with moderately low ceilings and prepare a pilot for landings in actual fog. This will make it possible for pilots to complete landings with approach

[3] The "over-run" is a stretch of pavement at one end of a runway which has not been built for regular use but which must be kept free of obstructions for emergencies.

lights when the weather has deteriorated and thus obviate the inconvenience of having passengers diverted to alternate landing fields at some distance from their destination.

HALF LIFE OF POLONIUM

Recent calorimetric experiments at the National Bureau of Standards have confirmed the value of the half life of polonium which was obtained in 1949 in the first accurate measurement of this quantity. In the NBS experiments,[1] an improved type of Bunsen ice calorimeter[2] previously developed at the Bureau was used to measure the heats of radioactivity of polonium samples over time intervals comparable with the half life. Values of half life computed from the observed decline in radioactive power were found to agree with the 1949 values within less than one part in a thousand. The investigation was carried out by D. C. Ginnings and Anne F. Ball of the NBS staff and D. T. Vier of the Los Alamos Scientific Laboratory.

Polonium, a radioactive element of atomic weight 84, was discovered in 1898 by Pierre and Marie Curie in the uranium-containing mineral, pitchblende. Because it is a convenient source of alpha particles, polonium has been used to a considerable extent in neutron sources and in various systems for removal of static charges during manufacturing processes. Its half life has been known approximately for some time but was not accurately measured until 1949, when W. H. Beamer and W. E. Easton[3] of the Los Alamos Scientific Laboratory obtained a value of 138.3 ± 0.1 days by observing a sample for 97 days. While their method was calorimetric, its principle was somewhat different from the NBS technique; they determined the rate of heat-energy emission by the sample by measuring the temperature difference between a container holding the sample and an identical container which was empty. Because the NBS ice calorimeter offered a convenient method for measuring the energy rates involved and might thus be expected to provide an accurate check on previously determined values, the Bureau developed a procedure for applying the ice calorimeter to this problem.

The NBS ice calorimeter measures heat input by weighing mercury. That is, the heat given off by the sample is allowed to melt ice that is in equilibrium with water in a closed system, and the resulting volume decrease is determined from the weight of mercury drawn into the

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[1] For further details see "Calorimetric Determination of the Half Life of Polonium," by D. C. GINNINGS, ANNE F. BALL AND D. T. VIER, *J. Research NBS*, Feb. 1953.

[2] "Heat Capacity of Sodium Between 0° and 900° C., the Triple Point and Heat of Fusion," by D. C. GINNINGS, T. B. DOUGLAS AND ANNE F. BALL, *J. Research NBS*, Vol. 45, p. 23 RP 2110 (1950).

[3] WILLIAM H. BEAMER AND WILLIAM E. EASTON, *J. Chem. Phys.*, Vol. 17, p. 1298 (1949).

system. Although the NBS ice calorimeter was not originally developed for determination of radioactive power, it has been found well adapted to measurement of the heat evolved by curie or multicurie quantities of alpha- or weak beta-emitting radioactive materials. One important advantage is the very small heat leak—of the order of 0.0002 watt. An additional advantage for experiments conducted over extended periods is the almost automatic operation of the calorimeter, which requires very little attention during the measurements.

In the NBS investigation, the heats of radioactivity of four samples of polonium, which initially developed power in the range of 0.17 to 1.4 watts, were measured with the ice calorimeter over a period of about seven months. The polonium samples were sealed in glass or metal vessels, which in turn were enclosed in brass outer containers made to fit closely in the calorimeter's central well—a thin-walled tube surrounded by the enclosed ice-water system and an outer ice bath. As the sample gave off heat, the resultant change in the volume of the ice-water system due to melting of the ice was transmitted to a mercury reservoir, resulting in a change in the amount of mercury in a weighed beaker. After a small correction for heat leak, the mass of mercury displaced was converted to an energy value by use of the calibration factor of the calorimeter, previously determined for the instrument by introducing a known quantity of heat electrically. From the measured duration of the experiment, the energy value was converted into average power. As the rate at which heat energy was produced by the sample was proportional to the number of atoms present, the exponential law of radioactive decay could then be used to determine the half-life of the sample. The best value thus obtained was 138.39 days ± 0.1 per cent, in close agreement with the value of 138.3 $\pm$ 0.1 reported by Beamer and Easton in 1949.

NOTES FROM THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

POLARIZED LIGHT: A RESEARCH TOOL

One of the most interesting activities of the Mechanical Engineering Division centers in the Halstead Laboratory where many important research projects are carried on with polarized light as the main tool. It is here that photoelastic analyses are done to observe the behavior of machine parts and other structures under stress.

Perhaps the most common application of photoelastic analysis is to certain problems that occur in the building and construction field. In construction work it is evident that each member of a structure must be big enough—have sufficient strength—to carry its load without breaking. However, engineering economy demands that the structure contain no more material than necessary. To determine the necessary strength (hence, size) sometimes requires very complicated calculations and, under certain conditions, the problem may defy mathematical analysis. Photoelastic analysis can be used to solve many of these mathematically indeterminate stress problems. Using the photoelastic method, an engineer is able to discover and measure stress concentrations which occur at such points as the corners of a machine part, or at similar points in other structures where considerably higher stresses occur than those existing throughout the structure generally, and where failure is apt to occur. The method can be used to analyze the stresses in any structure or machine part whose thickness is essentially constant.[1]

The principal part of the equipment is the Polariscope, in which parallel rays of polarized light pass through a model which is made of a transparent isotropic material that becomes bi-refringent when stressed, and which represents the machine part or structure under investigation. If this model is put under a load in the same manner as the real part, there appear within it patterns of light and dark bands. (If monochromatic light is used the patterns are white and gray; white light produces vari-colored patterns.) When properly interpreted, these bands indicate the stresses to which the structure is being subjected when under load. A great many of these bands close together signify regions of stress concentration, and these are the points at which the structure is likely to break, if overloaded (see Fig. 1).

Problems concerning shear stresses in soils under pressure from a building can often be solved by passing polarized light through a

[1] How this method was used to study pressure distribution in packing rings was discussed in this JOURNAL for March, 1952, p. 253.

model of a section of the foundation. In this case, the model is made of gelatin congealed between the glass plates of a tank. When stressed, gelatin also becomes bi-refringent and shows a stress pattern due for

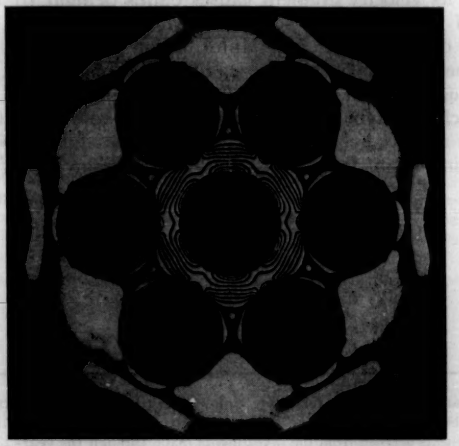


FIG. 1. Section of a high-pressure chamber, showing regions of equal shear stress when pressure is in the central chamber. Where dark bands are closest, stress concentrations are highest.

instance to the weight of a building or to subsurface cavities such as a tunnel. Critical shear stresses that cannot be determined in any other way can thus be clearly indicated.

FIG. 2. Air stream around a section of a B-29 wing at 18° elevation, as observed by Bentonite flow method.

A similar technique is used to study the flow of fluids. Here, the fluid is represented by a colloidal suspension of a mineral (generally Bentonite) in water. The solution flows through a tank that is built

with channels, vanes and obstructions to produce the desired paths for the fluid—again, in two dimensions. The nature of the mineral suspension is such that when it flows, and when polarized light is transmitted through it, its velocity gradient causes bands of light to appear along the lines of constant shear stress, allowing the observer to chart its flow, areas of turbulence, etc. Information of this sort is useful in designing heating ducts, chimneys, hydraulic machinery, nearly any device through which a fluid must flow. See Fig. 2.

With such a valuable and versatile tool at its command, the Halstead Laboratory already has a long history of successful projects, and has become an important factor in our industrial research program.

A. O. BERGHOLM

The Franklin Institute, a privately endowed institution which exists today only because of the generous benefactions of its supporters, is justly proud of the dividends in human benefits accrued from the investments made through the years by its friends. The accumulation of many bequests for general endowment and income to be used where it will do the most good, is the most important and pressing need of The Franklin Institute today.

Checks should be made payable to The Franklin Institute and sent to The Franklin Institute, 20th and The Parkway, Philadelphia 3, Pennsylvania.

Transfer of property should be made to The Franklin Institute.

A typical form is:

I hereby give and bequeath to The Franklin Institute of the State of Pennsylvania the sum of.....to be used as The Board of Managers of The Franklin Institute may think advisable for the benefit of said Institution. (For the principal and income thereof to be devoted to.....).

Additional information on gift and memorial opportunities may be obtained through the above address.

THE FRANKLIN INSTITUTE

MINUTES OF THE STATED MEETING

February 18, 1953

The Stated Monthly Meeting of The Franklin Institute was held on February 18, 1953, in the Lecture Hall. S. Wyman Rolph, President, called the meeting to order at 8:15 P.M. Approximately 400 persons attended.

The President stated that the Minutes of the December meeting were printed in full in the January JOURNAL. There being no corrections or additions, they were approved as printed. He stated further that the Minutes of the Annual Meeting in January were being printed in the February JOURNAL, which had not yet been mailed to members. Approval, therefore, was postponed until the next meeting.

The Secretary reported the following elections to membership for the month of January:

Active	26
Associate	27
Student	18

and a total membership of 6634 as of January 31, 1953.

The President then introduced the speaker for the evening—Dr. W. F. G. Swann, Director of the Institute's Bartol Research Foundation, and Senior Adviser to the Institute's Laboratories for Research and Development. Dr. Swann's illustrated lecture—"Is the Universe Planned?"—was enthusiastically received by the audience, and will be published in a future issue of the JOURNAL.

Dr. G. H. Clamer, President of Ajax Electro Metallurgical Corporation and member of the Institute since 1891, who had suggested the subject, thanked Dr. Swann for delivering the lecture in his honor. Dr. Clamer said that of the more than two hundred papers and lectures by Dr. Swann this had proved the best, a statement the audience endorsed by a round of applause.

The meeting adjourned at 9:35 p.m. with a rising vote of thanks to Dr. Swann.

HENRY B. ALLEN,
Secretary

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in the JOURNAL within the next few months:

KRZYWOBLOCKI, M. Z. v.: On the Fundamentals of the Boundary Layer Theory.
ARSOVE, MAYNARD G.: The Algebraic Theory of Linear Transmission Networks.
ZADEH, L. A.: A Contribution to the Theory of Nonlinear Systems.
LAUER, HENRI: Operating Modes of a Servomechanism with Nonlinear Friction.
MOON, PARRY AND DOMINA EBERLE SPENCER: Some Coordinate Systems Associated with Elliptic Functions.
COURT, ARNOLD: Wind Extremes as Design Factors.
ALDRICH, LELAND E.: Solution of Algebraic Equations.

MUSEUM

With summer approaching and bringing with it the menace of thunderstorms, visitors to the Museum should find interest in one of the new exhibits. This relates to lightning rods, and it is appropriately located near the Westinghouse Surge Generator, where private lightning storms are fashioned for the visitors.

It is difficult to obtain any reliable figures which would permit an estimate of the value of lightning rods, but the best recommendation for the use of Franklin's rod comes from the reports of fire insurance officials, who have paid particular attention to the monies they have have had to pay for losses from fire due to lightning. An over-all figure would not be satisfactory because lightning storms are not uniformly distributed and some districts get their lightning in much more malignant form than others. However, the State of Iowa enjoys the reputation of having more than the average number of storms, they are severe, and the number of rural buildings (where lightning is singularly effective) is high. One would naturally expect an unusually large proportion of fires caused by lightning under the favorable conditions presented. The number of fires due to this cause is not known, but out of 924 fires reported in one year only 50, or 5%, occurred in buildings which had lightning rod protection, and it is by no means sure that lightning was the cause of this small number of fires. It is estimated that one-half of rural buildings have lightning rod protection.

In spite of these figures lightning still takes a considerable toll of unprotected property in the country from which the rod came. The indifference on the part of property owners can be traced to several causes, one of which is the impression created by unscrupulous lightning rod agents of about 75 years ago, who profitted at the expense of a credulous public by erecting rods without regard to the principles which had to be followed. So that there may be no mistake about the correctness of the conditions for security, the new exhibit furnishes a diagram showing how the protection should be designed for a residence without going to the extreme of those who enclosed the Brussels city hall in a copper network.

The exhibit shows some very old lightning rods with terminations one very rarely sees today, but the exhibit is not historical. It shows how the main processes of the lightning discharge are built up in the thundercloud, or cumulus nimbus. Thunder-clouds usually develop from large masses or long banks of cauliflower-headed cumulus clouds, which are lower in height and smaller. Such a group of clouds amalgamates into one mass with an extensive upward development of towering heads and the formation of the threatening base. An idea is presented of the great dimensions of the thunder-cloud.

Some practical advice is given to those who may find themselves inadvertently exposed to the danger of lightning.

The exhibit is sponsored by the Home Insurance Company in honor of the Franklin Fire Insurance Company which was founded in Philadelphia in 1829 and is now amalgamated with the Home Insurance Company.

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the Library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A. M. until 5 P. M.; Wednesdays and Thursdays from 2 P. M. until 10 P. M.

RECENT ADDITIONS

AERODYNAMICS

BRAMBLE, A. YORK. Air-plane Flight. 1952.

ASTRONOMY

YALE UNIVERSITY. (ASTRONOMICAL) OBSERVATORY. General Catalogue of Trigonometric Stellar Parallaxes. 1952.

BOTANY

PRESTON, R. D. The Molecular Architecture of Plant Cell Walls. 1952.

CHEMISTRY AND CHEMICAL TECHNOLOGY

LE BRAS, JEAN. Elements de Science et de Technologie du Caoutchouc. 1951.
MACDOUGALL, FRANK HENRY. Physical Chemistry. Ed. 3. 1952.
PRICE, DONALD. Detergents. 1952.

DICTIONARIES

KINGZETT, CHARLES THOMAS. Chemical Encyclopedia. 1952.
NOEL, JOHN V., JR. Naval Terms Dictionary. 1952.

ELECTRICITY AND ELECTRICAL ENGINEERING

CARR, T. H. Sub-Station Practice. Ed. 2. 1952.
HARWOOD, PAISLEY B. Control of Electric Motors. Ed. 3. 1952.
KEMP, PHILIP. Alternating Current Wave Forms. Ed. 2. 1952.
RZIHA, E. V. Starkstromtechnik, Taschenbuch für Elektrotechniker. 1951.
STUBBINS, GEORGE WILFRED. Commercial a. c. Measurements. Ed. 3. 1952.

ELECTRONICS

KAMEN, IRA, AND WINNER, LEWIS. TV . . . FM Antenna Installation. 1949.
LOW, ARCHIBALD MONTGOMERY. Electronics Everywhere. 1951.

ENGINEERING

HEYWOOD, R. B. Designing by Photoelasticity. 1952.
MURRAY, WILLIAM M. Fatigue and Fracture of Metals. 1952.
SEELY, FRED B. AND SMITH, JAMES O. Advanced Mechanics of Materials. Ed. 2. 1952.
STEWART, DONALD ARNOTT. The Design and Placing of High Quality Concrete. 1951.

FOOD

SHERMAN, HENRY CLAPP. Chemistry of Food and Nutrition. Ed. 8. 1952.

MANUFACTURE

LITTLE, ARTHUR, AND MITCHELL, K. A. Tablet Making. 1949.

MARINE ENGINEERING

WOODS HOLE OCEANOGRAPHIC INSTITUTION. Marine Fouling. 1952.

MATHEMATICS

GIBBS, W. J. Tensors in Electrical Machine Theory. 1952.
SOMMERFELD, ARNOLD. Mechanics. 1952.
TIPPETT, LEONARD HENRY CABEL. The Methods of Statistics. Ed. 4.

MECHANICAL ENGINEERING

HILTON, B. RICHARD. Engineering Machine Shop Practice. Ed. 3. 1952.
MEHL, CURT. Die Evolventenzahnform der Stirnräder mit Geraden Zähnen. 1951.
RIEGEL, FRANZ. Rechnen and Spanabhebenden Werkzeugmaschinen. Vol. 1. 1951.

OPTICS

HERZOG, ALOIS. Handbuch der Mikroskopischentechnik für Fasertechnologen. 1951.
SCHOMERUS, FRIEDRICH. Geschichte des Jenaer Zeisswerkes 1846–1946. 1952.

PETROLEUM

STANDING, M. B. Volumetric and Phase Behavior of Oil Field Hydrocarbon Systems. 1952.

PHYSICS

BARRON, S. L. C. T. R. Wilson and the Cloud Chamber. 1952.
BRÜGEL, WERNER. Physik und Technik der Ultrarotstahlung. 1951.
PAGE, LEIGH. Introduction to Theoretical Physics. Ed. 3. 1952.
WALL, CLIFFORD N., AND LEVINE, RAPHAEL B. Physics Laboratory Manual. 1951.
WINANS, JOHN GIBSON. Introductory General Physics. 1952.
ZEMANSKY, MARK W(ALDO). Heat and Thermodynamics. Ed. 3. 1951.

PLASTICS

BEAU, MAURICE. La Caséine. Ed. 2. 1952.
GREAT BRITAIN. MINISTRY OF SUPPLY. Plastics. 1952.

RAILROADS

SIMPSON, CHARLES REGINALD HANDBURY, AND ROBERTS, F. BROWNE. Locomotives and Their Working. Vol. 1–2. 1952.
WEBSTER, HARRY. Railway Motive Power. 1952.

SCIENCE

SARTON, GEORGE. A History of Science. 1952.

SUGAR

DILLEWIJN, C. VAN. Botany of Sugar Cane. 1952.
FERRER, F. A. LOPEZ. Manual Práctico de Fabricación de Azúcar de Caña. 1948.
FERRER, F. A. LOPEZ. Manual Práctico de Maquinaria y Aparatos en los Ingenios de Azúcar de Caña. Ed. 2. 1949.
VANDERBILT UNIVERSITY. Bagasse Utilization. 1952.
VAZQUEZ, E. ANTONIO. Caña de Azúcar. 1951.

ACTIVE MEMBERS ELECTED AT THE MEETING OF THE BOARD OF MANAGERS, JANUARY 21, 1953

ACTIVE FAMILY

Joseph S. Bates	Mrs. Morris E. Leeds	Charles Eugene Moore
William Fausset Bruce, Ph.D.		W. R. Stinson

ACTIVE

Guy C. Bell, Jr.	Walter S. Farley, Jr.	George W. Richards, II
Robert E. Benn	Charles N. Herbert	Earl A. Smith, Jr.
S. P. Borbonus	Julius Hollander	William R. Thomson
Clair C. Campbell	John P. McCann	Lawrence M. Weldon
John R. Curtis	Howard M. Purcell	Fred Wolf, Jr.

ACTIVE NON-RESIDENT

Philip Garnick	Jacob Herrmann	Blair R. Yetman
	William H. Park	

NECROLOGY

Sebastian Bauer '46	Arthur W. Howe '84	William I. Schaffer '36
Harvey N. Davis '29	T. H. Morgan '38	Irvin Shupp '36
Paul A. Griffith '36	Frank H. Moss '38	Harry D. Siegfried '44

BOOK REVIEWS

ELECTRON STRUCTURES OF ORGANIC MOLECULES, by Lloyd N. Ferguson. 335 pages, diagrams, 16 × 24 cm. New York, Prentice-Hall, Inc., 1952. Price, $6.50.

Will man change nature and the universe? He seems to be well on his way. Physicists are busy adding new elements to those existing since the dawn of history. Chemists create organic compounds unheard of in nature. Since Wöhler successfully performed his synthesis in 1828, man has added a million new compounds to those with which nature has endowed him. What are the tools used in the creation of these wondrous chemicals, in the synthesis of the miracle drugs with whose help man controls and rules his surroundings?

To synthesize one must analyze. Only systematic studies reveal the correlations between structure and the properties of matter. This is the tenor of Professor Ferguson's volume which outlines the essentials of electron-structural analysis. Two representative groups of physical measures acquaint us with up-to-date forms of analytical thought. The simple tools, temperature, density, viscosity, etc. are followed by more complex physical properties in the electro-magnetic field, such as electron diffraction, spectra, and dipole moments.

The volume is expository and descriptive in nature. On first reading, the heterogeneous correlation analyses may at times appear confusing. But the author has in general succeeded in homogenizing an admittedly composite subject matter. A first discussion of chemical bonds includes the concepts of the ion-dipole bond, the intermolecular semipolar bonds in π-complexes, and the hydrogen bond. The following short chapters deal with covalent bonds, distances, angles, and the intramolecular forces. Forays into electro-magnetic field theory, finally, lead to a climaxing chapter on the structural analysis of the penicillins. The last of three chapters on dipole moments, resonance, and absorption spectroscopy is a real timely contribution. It includes conjugation of chromophores, a fascinating matter on which Professor Ferguson himself has done fundamental work.

It was probably a wise choice to limit the volume almost entirely to organic analysis. Only by inference does the author approach the vastly more complex question of synthesis. The correlations between physical properties and electron structure are treated exhaustively. This is evidenced by scores of tables and quantitative graphs, listing hundreds of compounds from the literature of the last twenty years. This fact, together with the more than five hundred references, make the volume a prized possession in the hands of all. Diagrams and structural formulas have been neatly reproduced. The author index, almost eight pages long, reads like a "Who's Who?" in physical and organic chemistry. This reviewer was pleased to find in it the name of Kasimir Fajans, some twenty years after having worked with him at the University of Munich. The subject index, however, is a bad compromise between the number of desirable and possible listings. It falls short of the general level of the book and makes the search for specific compounds tedious.

The reader is not told how to synthesize a universal dye, how to operate the infrared spectrograph, nor how to cure all of man's ills. But he gets a good view of the many facets of analytical thought which spell success or failure in problems of synthesis. And synthesis alone, in its many ramifications, is the tool that enables man to continue his self-appointed rôle of master of the earth. Professor Ferguson's volume illuminates the questions of analysis and should find its way to the desks of graduate students and researchers alike.

CARL HAMMER

FILTER DESIGN DATA FOR COMMUNICATION ENGINEERS, by John Henry Mole. 252 pages, tables, diagrams, 16 × 25 cm. New York, John Wiley & Sons, Inc., 1952. Price, $7.50.

Filter Design Data is a new publication for communications engineers by a British author. Its contents are more extensive than one would find in a handbook but less rigorous than the treatment given in a book on circuit synthesis such as Guillemin's *Communication Networks*.

The work is obviously the result of a career devoted to the analysis of filters, contrasting with the occasional reference which most electrical engineers have to this subject. For the latter purpose the handbook presentations are usually sufficient. However, to the specialist devoted to telephone circuitry, where the refinements and tolerances on element values have great economic importance, the treatment set forth in this book is appropriate. To one experienced in communications this volume is reminiscent of the original *Transmission Circuits for Telephonic Communication* by K. S. Johnson, although it is more thorough than the latter.

Approximately two hundred pages of the book are devoted to the analysis of band-pass and band-rejection sections. In all cases the treatment is very detailed and separate chapters are provided for the calculation of effective loss, design of terminal sections, Tchebycheff behavior and effects of dissipation. Extensive treatment is also given the effect of tolerances, or small changes in component values, through the use of weighting functions. The latter also includes a discussion of the statistical addition of various effects. The concluding portions of the book contain numerous tables of useful functions.

The book is composed largely of curves and the associated algebraic relations describing filter phenomena. The explanations in the text are brief, making space available for the extensive circuit and mathematical treatment of the subject. The viewpoint of the author is obviously that of the engineer who has strictly the communications use of filters in mind. Although filter networks are extremely important today in the design of servomechanisms and related electronic devices, the form of the data given in this volume is not convenient for application in that field. Here the results are usually worked out in terms of frequency, and the concept of the transfer function used in servo work is absent.

The meticulous treatment given to the subject of filters by this author will be appreciated by engineers practicing in the communications sciences, and the clarity of its pictorial and mathematical figures gives it a direct appeal. It is not too likely, however, that it will be of general interest to the electrical engineers who have drifted from communications into the broad areas of applied electronics.

C. W. HARGENS

CHEMISTRY OF THE METAL CHELATE COMPOUNDS, by Arthur E. Martell and Melvin Calvin. 613 pages, diagrams, tables, 16 × 24 cm. New York, Prentice-Hall, Inc., 1952. Price, $10.00.

Chemistry of the Metal Chelate Compounds was written to fill a vacancy in this expanding field of chemistry, a field which is finding connection with many other branches of chemistry. It was intended to aid those engaged in research activity involving chelate compounds.

The subject of chelates is considered under the general topics of detection, stability, effect of chelating agent and of metal ion, bond type, structure, catalytic effect, reactions involving the metal ion, and uses. These divisions are rather broad and general. There is considerable overlapping of material, some very similar material being covered in different sections of the book. These topics are preceded by an introduction in which basic definitions and principles are given in a very logical manner. Considering the subject, the book is well organized with each topic complete in itself.

The general plan of the book is to emphasize the structure and the stability of chelates rather than their formation and chemical properties. The discussion of stability is extensive and is appended by a fifty-page table of stability constants. The indirectness of the determination of chelate structure is emphasized, a number of indirect physical and chemical methods (some of which seem to be of dubious value) are needed to fix a structure. A good discussion with particularly complete examples is given of the only direct method of determining chelate structures, X-ray diffraction.

It should be mentioned that the book is extremely well illustrated and well indexed. Extensive references are given with numerous descriptions of basic and exemplary work. It shows the great amount of work done on chelates, a good bit of it being quantitative. In some cases it does, however, seem to degenerate to a listing of chelates known.

The resulting volume should adequately fulfill the original purpose. It should also be valuable as a reference book to students and to scientists interested in metal chelate compounds.

M. H. BORTNER

DARWIN: COMPETITION AND COÖPERATION, by Ashley Montagu. 148 pages, 14 × 21 cm. New York, Henry Schuman, Inc., 1952. Price, $2.50.

The author of this essay takes upon himself the task of showing that many of the ideas of the Darwinian school of thought were erroneous. To biologists this may seem rather unnecessary inasmuch as most modern biologists have long ago ceased to believe that Darwin presented the whole explanation of evolution. It seems needless to point out that there has been a great mass of data accumulated since the latter part of the nineteenth century which has led students of evolution to somewhat different conclusions than Darwin reached.

The author attempts to show that the principle of survival of the fittest was more a product of the thought of the time of Darwin than of a sound considered interpretation of the then available data. He also stresses the point that the attitude engendered by this idea has wrought a great deal of harm in the past and is still doing so. The author prefers to emphasize the importance of cooperation and various altruistic tendencies in biological systems. From this, it is evidently only a step to apply these ideas to our modern social set-up. Whether or not this extrapolation is justified is for the individual reader to decide. The point that the author makes, that cooperation plays an important part in evolution, is certainly well taken, but one does get the feeling that perhaps the author's views are as much colored by the thought of his own times as he supposes Darwin's to have been by his.

The essay is one which presents relatively little meat for critical study. Actual data are missing and Professor Montagu's arguments are supported principally by quotations of opinions of various people.

WARREN R. STINEBRING

COMETS AND METEOR STREAMS, by J. G. Porter. 123 pages, plates, diagrams, tables, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1952. Price, $5.25.

The second book of the International Astrophysics Series is Dr. Porter's *Comets and Meteor Streams*. It is a work which is hardly intended for the layman. The background of the book indicates that this was to be a contribution to the science and not a popular treatment.

Dr. J. G. Porter, Principle Scientific Officer of H. M. Nautical Almanac Office, presented a lecture at the Physical Society Conference in 1947 and this volume is the expanded version of the lecture. He has succeeded in adding details of much used and little understood elements of his subject.

The book is unfortunately a brief one of little more than a hundred pages, but in this compass Dr. Porter has tackled two topics which are unusual in a treatise of this type.

Chapter IV concerns perturbations which are usually dealt with only in an advanced text. Cometary orbits, as they are normally computed, are the osculating orbits in which the perturbative effects of the other bodies in the solar system are ignored. However, for a definitive orbit these must be included in the computations. In some eleven pages of text Dr. Porter details an exposition of the perturbations, computations and effects. This chapter is a magnificent bit of scientific exposition.

In Chapter VI the author deals with meteor orbits. This has also been a neglected topic. The only book with which this reviewer is familiar which devotes considerable space to these orbits is Dr. C. P. Olivier's *Meteors*.

The other five chapters of the book deal with the conventional topics such as comets, their statistics and the computation of their orbit; and finally, meteor velocities and streams.

Comets and Meteor Streams is an unusual type of book today, for it goes right back to basic celestial mechanics. The dynamics of the solar system are becoming more and more important today because of the great strides in cosmogonical studies. These researches are going forward at an increased tempo and for this reason alone the book should find a niche in the literature today.

Because of the tremendous interest in the origins of comets and meteors bodies the one disappointing feature of the book was the exceedingly brief reports on Dr. F. W. Whipple's and Dr. Jan Oort's work on the origin of the comets.

The author has ingeniously kept away from descriptive parts of the subject for it is in that field where his contribution could be least effective.

The bibliography contained at the end of each chapter is most welcome for those who would pursue specific details further.

Comets and Meteor Streams is a welcome contribution to the astronomical scene and deserves its place on the bookshelf beside the others dealing with the solar system.

I. M. LEVITT

ELASTICITY IN ENGINEERING, by Ernest E. Sechler. 419 pages, diagrams, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1952. Price, $8.50.

As mentioned before in these book-review sections, there have been many books on the subject of strength of materials or the theory of structures, and one often wonders why so many are published. Most of them are indistinguishable in subject matter from each other. Occasionally, however, a notable book such as *Elasticity in Engineering* does appear. Written by Dr. E. E. Sechler, Professor of Aeronautics at the California Institute of Technology, *Elasticity in Engineering* is what its title means: the practical use of the theory of elasticity in structural analysis in the fields of aeronautical, civil and mechanical engineering. The book is distinguished by a clear treatment of the fundamental theory of elasticity and the application of that theory to stable and unstable structures. As the author emphasizes, space did not permit exposition of all the methods of structural analysis; nevertheless, a surprisingly large number are covered. A student or structural engineer should find the book unusually informative.

The book is divided into three parts: Part I, Fundamental Equations and Analysis Methods; Part II, Engineering Problems in Stable Structures; and Part III, Engineering Problems in Instability. In Part I are found the basic concepts of the theory of elasticity including uniform and non-uniform stress conditions, strain conditions and the stress functions. The equilibrium and strain equations in Cartesian, cylindrical and spherical coordinates are given. Included also are the elastic energy principles. In Part II such problems as the theory of simple beams, continuous and redundant structures, torsion and stable plates and shells are discussed. In Part III such problems of instability as those encountered in the design of columns, beam columns and plates and shells are presented. Throughout Parts II and III, in the practical application of the theory, Dr. Sechler has given, in many cases, the precise solutions of problems, and, where these have been very cumbersome, has shown approximate solutions, which may be handled much more quickly, and how much they differ from the exact solution. Needless to say, many practical examples have been included to illustrate the various methods of structural analysis.

E. W. HAMMER

TEXTILE CHEMICALS AND AUXILIARIES, edited by Henry C. Speel. 493 pages, diagrams, 16 × 24 cm. New York, Reinhold Publishing Corp., 1952. Price, $10.00.

This book has been written from a practical viewpoint by twenty nine leading chemists and edited by a widely known expert in the textile field, and is of interest not only to those engaged in the textile industry, but also to the layman whose contacts are merely those of a consumer. Part I, comprising the first seven chapters, deals with the nature and processing of fabrics; the second part, including sixteen chapters, covers the raw materials and auxiliaries used in fabric processing.

In Part I, the reader is introduced to various types of fibers, both natural and synthetic, with a brief explanation of their physical and chemical properties, and the different steps necessary in the preparation of the raw fibers for manufacture into yarn or fabric. Several chapters are devoted to the fascinating processes of dying and printing fabrics. In the chapters on finishing, the reader is brought up-to-date by a discussion of recent advances in special coatings which render fabrics crease-resistant, water proof or moth proof. The information is clearly presented, all terms defined for the benefit of the uninitiated and the fundamentals of the chemistry involved simply explained.

In Part II, each class of materials used in the processing of fibers and fabrics, as for example, fats and oils, starches, gums and glues, solvents, is treated in more detail. In general in each chapter, the author briefly introduces his subject, gives the historical background, explains the type of the auxiliary under discussion, its chemistry, its application to the textile industry

and the methods employed in its use. For those desiring a more comprehensive treatment of the subject, there are good bibliographies and references at the end of each chapter.

The book is well indexed. Throughout the book, the material is well organized so that it is not only a readable book for anyone interested in the subject, but would serve well as a reference.

RACHEL G. FRANKLIN

SUPERCONDUCTIVITY, by D. Shoenberg. Second edition, 256 pages, diagrams, 15 × 22 cm. New York, Cambridge University Press, 1952. Price, $6.00.

The first edition of Schoenberg's *Superconductivity*, published in 1938 as a Cambridge Physical Tract, was at that time an excellent summary of the existing knowledge of superconductivity. The rapid advances in both experiment and theoretical understanding of the subject since that time, however, have resulted in the need for a more up-to-date treatment. The second edition, published in the Cambridge Monographs series, satisfies this need very well.

The book is divided into six chapters dealing, after a general introduction, with the magnetic properties of superconductors, the thermodynamic and thermal properties, the surface penetration of a magnetic field, the intermediate state, and, finally, the recent theoretical work. The discussion throughout is well-documented with experimental data in the form of graphs and tables. In addition, the appendix provides a useful list of transition temperatures and critical fields for the known superconducting elements, alloys, and compounds. Another appendix consists of notes on recent work which appeared too late for inclusion in the text.

Its comprehensive coverage of the subject, together with the inclusion of some four hundred references to original papers, should make this book a valuable addition to the library of student and research worker alike.

D. P. DETWILER

DEVELOPMENT OF THE GUIDED MISSILE, by Kenneth W. Gatland. 133 pages, illustrations, 14 × 22 cm. New York, Philosophical Library; London, Illiffe & Sons, Ltd.; 1952. Price, $3.75.

Books on interplanetary travel are appearing in confusing profusion but those containing information on the basic research projects which will show the way to this adventure are few in number.

This reviewer knows of only two books in English which contain comprehensive information on guided missiles. One is the old reliable standby by Willy Ley, *Rockets, Missiles: Rockets and Torpedoes*. Now comes another book on guided missiles by one of England's bright lights in the space travel field—Kenneth W. Gatland.

Mr. Gatland is one of the chief spokesmen and pioneers of the British Interplanetary Society, a group which has done much to advance the cause of travel beyond the atmosphere of the earth. His contributions which have appeared in the *Journal of the British Interplanetary Society* are remarkable for their originality and thoroughness of treatment. Mr. Gatland adds to his reputation with this book.

The author is preaching a crusade. He believes, as do many of his colleagues, that the British Isles are vulnerable to attack from the mainland by guided missiles and his voice is raised against the complacency of the military in not forwarding the progress of weapons to counter the blows which might fall from the air.

He has collected all the unclassified information on the evolution and present state of the guided missile. This has been assembled in a clear, concise form for the enlightenment and enjoyment of both the technical and the lay reader. His exposition of the problems of developing directed rockets is remarkable for the quantity and quality of information collected and concentrated into a thin book of some 133 pages. It is remarkable for its completeness. For example, nowhere else is such a good tabulation of the characteristics of all guided missiles to be found.

As would be expected from a prolific contributor to the *Journal of the BIS* the last chapter of the book is devoted to space travel. There is a propaganda theme which is sensed as one concludes the book. The members of the BIS, including this reviewer, are all space-travel conscious and are determined to gain wide acceptance of their ideas by the public. One good

method to secure action is to tie the proposal to something of the utmost urgency—national defense. Mr. Gatland shows himself exceptionally clever in urging the adoption of a huge guided missile program as a defense measure when his basic aim is clearly space travel.

Development of the Guided Missile is a sound and significant contribution to space travel literature for, in addition to a clear perception, the author writes with a sincerity and vigor which comes of years of close association with his subject. I. M. LEVITT

PHOTOCONDUCTIVITY IN THE ELEMENTS, by T. S. Moss. 263 pages, diagrams, 15 × 22 cm. New York, Academic Press, Inc., 1952. Price, $7.00.

The recent development of the transistor has served to focus a great deal of interest, both theoretical and experimental, upon the properties of semi-conductors and insulators. Since photoconductivity measurements provide a useful tool in the study of these properties numerous workers, including the author, have undertaken to study its occurrence and characteristics in these materials.

Somewhat more than one quarter of the present volume is devoted to a survey of the theory of photoconduction and closely related matters. The remainder is concerned with the experimental details and results obtained by the author and by previous workers in their investigations of photoconductivity in boron, diamond, silicon, germanium, grey tin, phosphorous, arsenic, antimony, sulfur, selenium, tellurium, and iodine.

This book contains a considerable quantity of previously unpublished data on photoconductivity, along with a good description of the experimental details. However, it is the feeling of the reviewer that the subject matter is perhaps more appropriate to a research paper than to a book. D. P. DETWILER

SOLUBLE SILICATES, Vol. 2, by James G. Vail. American Chemical Society Monograph No. 116. 669 pages, illustrations, 16 × 24 cm. New York, Reinhold Publishing Corp., 1952. Price, $10.00.

The second volume of the two-volume monograph written to replace the out-of-print *Soluble Silicates in Industry* (ACS Monograph No. 46) is a most comprehensive survey of the diversified and multitudinous industrial applications of the soluble silicates. Its organization is excellent. The vast number of applications and methods have been organized into five large chapters, each of which treats a general type of application. These chapters in turn are subdivided into logical divisions which discuss specific applications. A short discussion of the principles involved precedes the descriptions of the uses and methods and enables the reader to understand better the actual physical processes involved.

The first chapter discusses the role of soluble silicates as a soap builder and a component of detergent mixtures. A variety of mixtures are described, and the advantages obtained by using the silicates are shown by means of plots and illustrations generously used. The chapter is further logically divided into applications at inorganic, and at organic surfaces. The specific applications described are many and varied, and generally cover almost all of industry.

The other general chapters are "Coatings and Films," "Bonded Surfaces," "Sols, Gels, and Polymers in Industry," and "Physiological Behavior." These chapters describe the role of soluble silicates in such diverse applications as adhesives, cements, insulation, molded products, polishing wheels, soil stabilization. The use of silica gels and sols is of significant industrial interest, and is treated in some detail. Present methods of production are described as are the many uses of this important material. Because of the substantial tonnage and widespread use in industry, a chapter is devoted to the physiological behavior of soluble silicates.

This book can be recommended for both lay and technical personnel in view of its excellent organization, its readability, and wide application of the products it describes.

S. MUCHNICK

BOOK NOTES

ELEMENTS OF FOOD ENGINEERING, Volume 1, by Milton E. Parker, E. S. Harvey and E. S. Stateler. 396 pages, illustrations diagrams, 16 × 24 cm. New York, Reinhold Publishing Corp., 1952. Price, $8.75.

This new series, which comprises three volumes, covers all aspects of food engineering. Volume 1 is devoted to the engineering factors involved in food processing. It describes in detail the agricultural and nutritional aspects of food production, the extent of the food processing industries, major classifications of foods and refined foods processing. The book can be used as a text for college courses in Food Technology, or as a guide to those already engaged in food processing. Even the lay reader can find much of interest, as he reads how spices are extracted, how catsup is bottled, how coffee is cured, how green tea is produced. The text is full of interesting historical notes and modern illustrations, both of which add considerably to the readability of the text.

METAL DATA, by S. L. Hoyt. Second edition, 526 pages, tables, diagrams, 18 × 27 cm. New York, Reinhold Publishing Corp., 1952. Price, $10.00.

Metal Data is the title of the second edition of the author's *Metals and Alloys Data Book*, published in 1943. The second edition has been extensively revised and enlarged to include an additional 200-odd pages of text, 225 extra tables, and 113 more illustrations. New data on heat treated steels have been incorporated, and the treatment of the "super-alloys" has been brought up-to-date. New materials are included only when reliable data are available. This second edition of a widely used reference work should be even more valuable than its predecessor.

QUANTITATIVE ANALYSIS, by Harvey Diehl and G. Frederick Smith. 539 pages, diagrams, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1952. Price, $5.00.

Intended for use in the sophomore year, this new text gives the elementary principles and practice in quantitative analysis, with a prerequisite of general chemistry only. Placing about equal emphasis on theory and practice, the text stresses a sound knowledge of descriptive chemistry. The text should prove easy to teach, since working directions for many of the standard methods are given in sufficient detail to make supplementary instruction unnecessary. Colorimetry is treated in a separate chapter, given equal rank with gravimetric and volumetric analysis. A supplementary answer book may be obtained, giving not only answers to problems, but also the principal steps and numerical data involved in the solution of the problems.

THE PRINCIPLES OF PHYSICAL METALLURGY, by Gilbert E. Doan. Third edition, 331 pages, illustrations, diagrams, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1953. Price, $5.50.

Although essentially the same book as the popular previous editions, this new version of *The Principles of Physical Metallurgy* contains several innovations. Chapter III has been rewritten to cover the explanation of the physical properties of metals in terms of electrons and atoms, and to bring up-to-date the discussion of deformation and annealing. Also, new casting techniques have been added, including investment casting and directional solidification methods for sand casting. New testing methods for supersonic testing, brittle lacquers and stain, are discussed.

EFFICIENT PICKLING WITH "RODINE," Bulletin No. 13, American Chemical Paint Co. Revised edition, 80 pages, illustrations, 15 × 23 cm. Ambler (Pa.), American Chemical Paint Co., 1953.

A revised edition of *Efficient Pickling with "Rodine"* has just been issued by the American Chemical Paint Co. as Bulletin No. 13. Covering all aspects of this phase of iron and steel manufacture, the booklet gives suggestions for the use of the company's selective inhibitor "Rodine," in the pickling of sheets, plates, strips, forgings, bars, stainless steel, castings, etc. Introductory sections discuss pickling in general, scale formation, cleaning of metals before

pickling. The chapter on "Rodine" defines this product, gives results of laboratory tests on it, lists the advantages of using it. The booklet is available on request to the Ambler offices, for those who are interested.

PHYSICS FOR SCIENCE AND ENGINEERING STUDENTS, by W. H. Furry, E. M. Purcell and J. C. Street. 694 pages, diagrams, 18 × 26 cm. New York, The Blakiston Co., 1952. No price.

In the preface of this book, the authors state their aim to be the accurate, clear yet simple presentation of basic physical principles, without complicated or involved discussions, a goal which, for the most part, they appear to have achieved. More than enough material for a typical one-year college course in elementary physics has been included, leaving a certain amount of selection to the judgment of the teacher, thereby increasing the usefulness of the book.

The use of calculus has purposely been kept to a minimum, especially in the beginning of the book. The many clear diagrams, the arrangement of the text with two columns to the page, the simplicity of the language, all contribute to ease in reading and understanding. It is regrettable, however, that the chapters and paragraphs are not noted on each page to help the reader locate quickly the cross references which frequently appear throughout the text.

ELECTRONIC ENGINEERING PRINCIPLES, by John D. Ryder. Second edition, 505 pages, illustrations, diagrams, 15 × 22 cm. New York, Prentice-Hall, Inc., 1952. Price, $6.75.

In revising this book, the author has been able to add much new and timely material without greatly increasing the length of the book. This has been accomplished by condensing some of the original material and consolidating some subjects, to make room for fuller discussions of amplifiers, feedbacks, etc. A new chapter on solid-state electronics has been added with a section on the transistor. The revision is further enhanced by the inclusion of new problems and new illustrations for nearly all chapters.

V.2
195

ANALYTIC MECHANICS, by Virgil Moring Faires and Sherman Daniel Chambers. Third edition, 540 pages, diagrams, 16 × 24 cm. New York, The Macmillan Co., 1952. Price, $6.00.

The most readily noticeable change in this third edition of a standard text is the inclusion of numerous historical and biographical sketches, which add interest to the presentation. The only new mechanics topics are those in the appendix and in the chapter on vibrations, although many new illustrative examples have been added throughout the text. Another feature is the repetition of important basic ideas, in an effort to impress these upon the students, without boring the teacher.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, DIRECTOR

Seminar on Cytological Phenomena in Ascites-Sarcomas of Rats (illustrated by motion pictures).—DR. SAJIRO MAKINO of the Zoological Institute, Faculty of Science, Hokkaido University, Sapporo, Japan, addressed the Seminar of the Biochemical Research Foundation on Tuesday afternoon, October twenty-first.

I take a great pleasure in receiving this invitation to give a talk about our cytological studies of cancer. I am an animal cytologist, and during the past twenty years, I have engaged in the study of animal chromosomes relating to the sex-determination, and on evolutional changes of chromosomes. I have had particular interest in cancer problems in connection with cytology. At present I have been working on cancer cytology, with the ascites-tumors developing in rats as material.

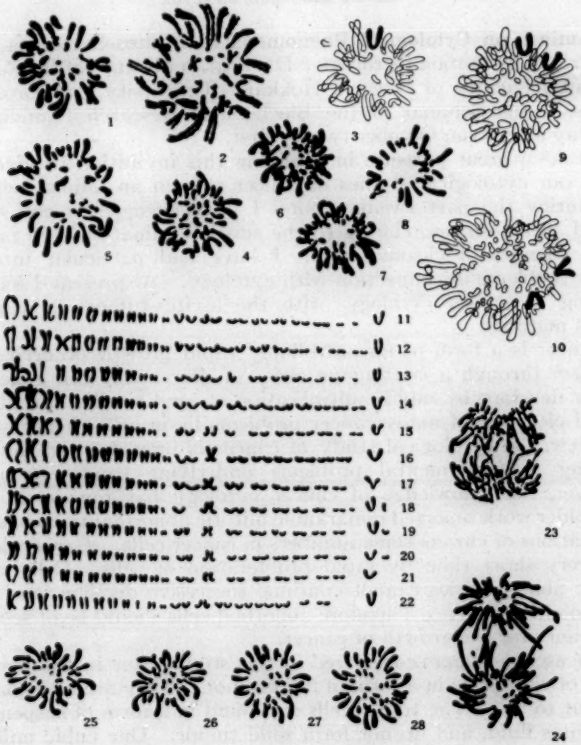
Cancer is a form of uncontrollably rapid growth occurring in an organism through a continuous series of descendent cell generations. Cancer develops by rapid multiplication of cancer cells, so that fundamental elements of many cancer problems lie in cells themselves. In this view, the cytological study of cancer is important to the understanding of fundamental problems underlying cancer phenomena. However, our knowledge of cancer cytology has remained meager. Most older work reported remarkable mitotic abnormalities and striking abberrations of chromosome numbers in cancer cells. Cancer develops in a very short time by rapid proliferation of cells. Cells showing mitotic abnormalities cannot continue successive division, and degenerate sooner or later. Therefore abnormal cells should be of secondary importance for the growth of cancer.

The ascites-tumors considered in this study, grow in the peritoneal cavity of white rats in a form of fluid tumor. They are transplantable from rat to rat. The tumor cells are found in a form of suspension in the ascites fluid, and do not form solid tumor. One cubic millimeter of tumor-ascites contains about one million tumor cells. The tumor cells are clearly distinguishable from blood cells by their characteristic features. By injecting a small amount of tumor-ascites by means of a fine pipette into the body cavity of the new rat, the tumor cells multiply and start the formation of new tumor. The tumor rats die on an average of from 10 to 13 days.

At present, we have kept in our laboratory five different kinds of ascites sarcomas. They all develop in white rats. They are (1) Yoshida-sarcoma, (2) MTK-sarcoma I, (3) MTK-sarcoma II, (4)

Hirosaki-sarcoma, and (5) Takeda-sarcoma; the first three were experimentally induced, the last two are of spontaneous origin.

Generally, the tumor tissue shows remarkable mitotic abnormalities. The ascites-tumors here concerned are not exceptional in this



Chromosomes of the Yoshida-sarcoma: 1–8, metaphases of tumor strain cells, having subdiploid chromosome complexes; 9, subtriploid cell; 10, subtetraploid cell; 11–22, serial alignments of paired chromosomes in descendant order, from tumor strain cells. 23–24, anatelophase of tumor strain cells, the division of chromosomes being almost regular. 25–28, metaphase chromosomes of ordinary tissue cells.

respect. Many older workers have considered that the abnormal mitoses cause the malignant growth of the tumor. Our recent findings are, however, contrary to this opinion.

In each of these types of ascites tumor, a group or groups of cells

were found which divide with regular mitotic behavior and possess a constant and characteristic chromosome complex. These are the stem line, or lines, of tumor cells which play an important role in the formation and growth of the tumor. These cells which behave in a regular mitotic manner are called the stem cells of the tumor.

The stem cells possess their own characteristic chromosome complexes. They multiply with regular mitotic behavior, and therefore remain unchanged in their chromosome individuality through successive transplant generations. The continuity of the tumor in successive transmission is due to the presence of these stem cells, and the later growth of the tumor is caused by their proliferation. In the course of multiplication, some of the stem cells become abnormal, due probably to various pathologic causes. Thus, the cells showing mitotic abnormalities are no other than the derivatives of the tumor stem cells. It is apparent that the cells showing abnormal mitoses cannot continue division much longer, and degenerate sooner or later. Therefore, they cannot contribute much to the growth of the tumor. Hence, the occurrence of the stem line of tumor cells is very significant.

The comparison of the chromosome constitutions of the tumor stem cells with those of the ordinary tissue cells is very interesting. In every kind of ascites-sarcomas, except the Takeda-sarcoma, the stem cells are characterized by having subdiploid chromosomes, 40 or thereabouts in number. In the Takeda-sarcoma the stem cells are tetraploid, being 80 or thereabouts in chromosome number. In every case, the chromosome complex consists of two groups; one group contains rod-shaped chromosomes, and the other group includes V- and J-shaped chromosomes. The five kinds of ascites-tumors here concerned are characterized by different numbers of rod-, V- and J-shaped chromosomes, as shown in Table I.

TABLE I.

	Type Cell	Rods	V's and J's
Yoshida-sarcoma		22–24	16–18
MTK-sarcoma I	1 V	24–26	15–17
	2 V	23–25	16–18
MTK-sarcoma II		28–30	11–13
	1 V	26–27	13–14
	2 V	23–26	14–17
	3 V	20–24	16–19
	4 V	19–22	17–20
	5 V	18–21	17–19
Takeda-sarcoma		Tetraploid (±80)	

Interestingly, changes in frequency of occurrence of the two cell types of the MTK-sarcoma I, and again of the five cell types of the Hirosaki-sarcoma were observed after successive transplantation. In the MTK-sarcoma I, the proportion of the 2V-type cell to the IV-type cell was approximately 3:1 in the 20th transplant generation. By the 50th generation the proportion of the 2V-type cell to the IV-type cell became about 1:3.5. In the Hirosaki-sarcoma, the frequency of the five types of cells observed in the 14th transplant generation is shown in Table II. In later generation, the 40th, the frequency of those cells showed a pronounced change as seen in Table III. From this evidence, it is certain that there is a difference of activity among various types of tumor cells.

The rod-shaped chromosomes occurring in tumor cells are morphologically identical to those of the ordinary rat cells. On this basis, they are probably of host (rat) origin. But, the V- and J-shaped chromosomes have become transformed, because they are not to be found in the host cells. They are specific to the tumor cells. The origin and nature of these two-armed chromosomes are unknown. But, evidence found

TABLE II.

Type Cell	Per Cent
1 V	3.0
2 V	17.9
3 V	20.9
4 V	49.2
5 V	9.0

in the Hirosaki-sarcoma suggests that the formation of some large V-shaped chromosomes is due to the fusion of two large rod-shaped chromosomes at their inner ends, since the number of large rod-shaped chromosomes decreases with the increase of large V-shaped chromosomes.

As an explanation for the origin of the particular chromosome constitutions occurring in the tumor stem cells, the evidence of chromosomal mutation during the transformation of an ordinary cell, or cells, is most plausible. Probably, at some stage during experimentation with carcinogenic agent, or in the life history of an individual cell, a mutational change took place in the chromosomes of one or more ordinary tissue cells. Perhaps, the process of mutation has not essentially disarranged the elements of cell metabolism or of mitotic activity, but enabled the mutated cells to grow autonomously. These cells multiplied to produce a stem line, or lines, of tumor cells having characteristic chromosomes. The occurrences of two types of stem cells in the MTK-sarcoma I, and of five types of such cells in the Hirosaki-sarcoma, lead to the supposition that mutational change in chromosomes

might have taken place not in a single but in several tissue cells of the original animal. And further, the fact that pronounced changes occurred in frequency of stem cells (Tables II and III) is suggestive that, in the course of the transplant generations, some inactive mutated cells underwent degeneration, and a group or groups of active cells having a well-balanced nucleus-cytoplasmic relationship persisted to produce a stem-line, or lines, of tumor cells.

The individuality or constancy of the chromosomes in the tumor stem cells has persisted unchanged in the case of heteroplastic transplantations. The ascites-tumors here concerned are specific to white rats (*Rattus norvegicus*) alone; in the body cavity of other kinds of rodents the tumor cells cannot exert malignant proliferation and degenerate sooner or later. But, the tumor cells inoculated into the black rats (*Rattus rattus*), house mice (*Mus musculus*), field mice (*Apodemus geisha*), voles (*Evotomyd bedfordiae*) and guinea pigs (*Cavia cobaya*) can divide for a short time. In division they show their particular chromosome constitutions specific to the tumor stem cells.

By the application of some chemicals, the stem cells also remain without visible evidence of damage. Injection of 0.1 per cent crude

TABLE III.

Type Cell	Per Cent
1 V	6.9
2 V	83.3
3 V	9.7

podophyllin in isotonic glucose solution resulted in damage to the large number of tumor cells, but a certain number of the stem cells have remained unaffected by the drug. Upon removal of the action of the drug, the stem cells begin to divide, and cause the regrowth of the tumor. Three successive injections were attempted at intervals of six days, and this induced a considerable prolongation of the life of tumor-bearing animals. The average life-span of ascites-tumor rats ranges from 10 to 13 days, while eight treated animals lived for 38, 41, 42, 42, 47, 49, 51 and 56 days, respectively. But the complete recovery of an animal from the tumor could not be obtained in the dosage used. It may be realized only by the complete destruction of all tumor stem cells. Higher dosage or over dosage are lethal to the tumor-bearing animal. A similar result was obtained by the applications of $CaCl_2$, $AlCl_3$, and H_2O_2. A question why some of the stem cells should remain unaffected by the chemicals is unanswered. This is one of the important subjects of investigation in cancer therapy. The resistant stem cells are generally small in size, being characterized by a small amount of cytoplasm and a compact nucleus. Probably, the resistant form of

cells is closely connected with the change of the mechanism in the surface of cells.

The mitotic events were observed through the course of division in a living cell with the aid of the phase-contrast microscope. Generally, the duration of all the division stages in the tumor cells is strikingly short as compared with that of the first division of grasshopper spermatocytes. For instance, it is 285 minutes at 30° C. in Chorthophage (Ris, H., *Biol. Bull.*, **96**, 90 (1949)) and 420 minutes at 20° C. in Podisma (unpublished data), while the duration of division stages in the Yoshida-sarcoma cells was observed to be 191, 155 and 70 minutes at 27°, 30° and 35° C., respectively. There is evidence regarding relations between the variation of temperature and the length of mitotic phases. The data presented in Tables IV and V indicate that in the tumor cells of

TABLE IV.—*Duration of Mitotic Phase in the Yoshida-Sarcoma Cells of 15 to 16 Microns in Diameter.*

Phase	At 27° C., min.	At 30° C., min.	At 35° C., min.
Prophase	34	20	14
Metaphase	105	80	31
Anaphase	8	5	4
Telophase	44	50	21
Total	191	155	70

TABLE V.—*Duration of Mitotic Phase in the MTK-Sarcoma I Cells of Various Diameters, Observed at 30° C.*

Phase	±13μ, min.	±15μ, min.	±17μ, min.
Prophase	12	12	20
Metaphase	54	62	125
Anaphase	6	5	12
Telophase	18	25	23
Total	90	104	180

similar dimension there is a remarkable decrease in the duration of each mitotic phase with rising temperature. This decrease is especially striking in metaphase and anaphase. Further, at constant temperature the duration of the mitotic phase is longer in the cells of larger dimension (Table V) than in the small tumor cells.

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CURRENT TOPICS

X-Ray Image Amplifier for Medical Use.—Radiologists and physicians now have a new tool in the X-ray image amplifier—a device that will give them a 200 times brighter view of their patient's internal organs in living action. An attachment for standard fluoroscopic equipment, the image amplifier is now going into production at the Westinghouse X-ray Division in Baltimore, Maryland after eight years of intensive research. Basically, the amplifier consists of a high-vacuum tube that electrostatically focuses and accelerates an electron stream.

The conditions confronting the physician in present-day fluoroscopy are comparable to those encountered in finding a seat in a movie after leaving the bright sunshine. To see the fluoroscopic image, the radiologist has to dark-adapt his eyes for at least 20 minutes. Now, with the image amplifier attached to his equipment, the physician can step into his dark room and see at once a bright image. He may still have to dark-adapt his eyes, though for only three or four minutes, before he can see all the details. What he sees after that will be a clear, easily discernible presentation of his patient's organs at work.

The new image amplifier increases the brightness of the fluoroscopic image after the X-rays have passed through the patient. This solution to the problem is necessary because the X-ray intensities are already approaching the patient's tolerance level. Since fluorescent screens have a gross efficiency of about 5 per cent, even a theoretically perfect screen would be only about 20 times as bright, and screen improvement alone cannot achieve the desired brightness gains.

Increased brightness of the X-ray image has been attained by converting the X-ray energy into light with a fluorescent screen, and thence to electrons by means of an adjacent photoelectric surface. These electrons are accelerated by a high potential placed across the vacuum tube, giving a brightness gain of 10 or more. A further gain is attained by electrostatic focusing of the electron stream to reduce the image to approximately one fifth its original size. The reduced image, made up of high speed electrons, impinges on a phosphor output layer that converts the electron stream back to a visible image, now brightened 200 times or more. As a final step, the intensified image is magnified by means of an optical system without loss of the increase in brightness.

The intensification achieved by reducing the image size in the electron-optical system is possible because the brightness is increased in inverse proportion to the area. This is so because all the electrons are utilized in forming the image. When the area is reduced, the total energy therefore remains constant. Thus the energy per unit area, which is proportional to brightness, must go up. Limitations of the optical magnification that can be obtained without loss of brightness or limitation to the freedom of position of the observer make it unprofitable to reduce the electron image to less than one fifth of the size of the X-ray image. The reduced image seen through an ordinary optical

magnifier appears in its original size with negligible loss of the brightness gained in the electron-optical reduction.

Fluoroscopic X-ray image brightening has been considered of such importance to the field of medical diagnosis that Westinghouse has made reports from time to time regarding the progress of experimental models for medical application. Fundamental development of the tube was done by the Westinghouse Research Laboratories. Once the theoretical aspects were mastered, the problem was given to the Electronic Tube Division to develop production methods and facilities for manufacturing the tube. This original production model is still in operation without any deterioration. Since this time, efforts have been devoted largely to the problems of consistently mastering the difficult techniques of manufacturing the tube.

The problem of taking this scientific achievement and converting it into a clinical tool has been accomplished by the X-Ray Division at Baltimore, Maryland. The problem of allowing the radiologist the freedom he is used to in rapidly moving the present-day fluoroscopic screen, as for example in watching a patient swallow a special meal and observing this meal drop through the esophagus and into the stomach, led to the development of a dual optical system. This optical system is used for rapid survey work with a magnification of 3×, giving the radiologist a large "exit pupil," or maximum freedom in the position to which he must bring his eyes. When the finest detail is to be observed, moving a simple lever converts the optical system to give 7× magnification.

Another feature of the optical system that was developed as a result of the clinical trials is the positioning of the image so that the radiologist can move in close to his patient in order to manipulate the patient, while at the same time viewing the X-ray image.

Other requirements that have been met to make this a real clinical tool are the inclusion of a film device for making radiographs for record purposes, and a phototimer for automatically timing radiographs.

The radiation hazard to radiologist and patient is basically the same as with conventional fluoroscopy, but there are two ways in which significant reductions of the hazard are possible. First, the radiologist may reduce the intensity of the X-ray beam, for example to one fourth the usual amount. Under these conditions he will still have an image 50 times (or more) brighter than the conventional. Second, because he learns what he wants to know so much quicker, the examination time and hence the radiation exposure is greatly reduced.

The future possibilities for fluoroscopy with the addition of amplification are manifold. In addition to shorter examinations, such techniques as stereofluoroscopy and even the televising of fluoroscopic images may become practical. All these possibilities are in addition to the great advantage that the physician will be able to perceive objects that are presently indiscernible.

Now commercially available, the image amplifier can be used as an attachment to present-day X-ray fluoroscopic equipment. Since all the work is done after the X-rays leave the patient, the new development will in no way affect the basic design or operating principles of equipment now in use.

New Plastic Scribing Process.—One of the worst bottlenecks in map production in the field, the color separation of map manuscripts, may be overcome by the plastic scribing process now being tested for military application by the Engineer Research and Development Laboratories, Fort Belvoir, Va.

In preliminary tests at the Laboratories the process has been found to reduce by about 50 per cent the amount of time required to extract from a crude map manuscript the detail that is to appear in different colors on the final map.

The process is similar to the glass-plate drafting method originally developed by the Coast and Geodetic Survey and used by many mapping agencies. In it, however, the glass is replaced by plastic sheeting. It consists of scribing with special tools the reversed blue line image of a map printed on a vinyl plastic sheet which has been coated with a fine pigmented paint. The details are traced on separate sheets according to the color in which they are to appear on the final map. Those to appear in black are traced on one sheet, in brown on another, in blue on a third, etc. From these sheets contact positives on film are made to which certain map symbols and lettering are added. By simple photographic processes lithographic plates used in the printing press are obtained.

Because the plastic scribing process eliminates ink-drafting and reduces color separation to a simplified tracing operation it can be done in one-half the time required by the present ink drafting method. An unskilled man can be trained in one week, while from three to six months of training are necessary for ink drafting.

The line work resulting is comparable to that achieved by the ink method or by glass plate drafting. The latter was also tested for military application but was not considered for field topographic use because of handling difficulties.

In the evaluation tests of the plastic scribing process, technicians at the Engineer Research and Development Laboratories worked with a swivel and a rigid graver designed for glass plate drafting. To eliminate difficulties encountered in their use a single tool which performs the functions of both was designed. This is a graver with a swivel point which can be made rigid. A sharpener to provide special points was also designed.

Tests of the plastic scribing are continuing with a view toward further improvement of the process.

What Causes the "Itch" in Poison Ivy?—A Columbia University chemist, Professor Charles Dawson, has moved another step closer to an understanding of the rash-producing action of poison ivy. He has isolated the toxic agent in the sap of that plant and broken it down into four distinct parts—one a saturated (solid) compound and the other three unsaturated (liquid) ones. Having identified the chemical structure of these components, Professor Dawson has recently been successful in synthesizing two of them, which are to be tested clinically to learn how they operate. Dr. Dawson believes that one or more of the four toxic elements which he has isolated may act as the sensitizer, preparing the poison ivy victim for the onslaught of the other itch-producing elements. If this proves to be the case, it may be possible to synthesize a range of compounds similar to the sensitizing agent and use these

to immunize or de-sensitize potential sufferers. Dr. Dawson points out that newborn babies, Eskimos, and Europeans are insensitive to the plant poison, which means that some preliminary contact with poison ivy is necessary to render people sensitive to it.

The Lederle Laboratories' division of American Cyanamid Company is supporting the research project, which may very well have world-wide importance if successfully concluded. Though poison ivy is indigenous to this continent, its relatives are found all over the world. In Japan, for example, the Lac tree, which provides the raw material for lacquer, causes a similar itch. A Japanese chemist investigated the toxic substance in this sap in the early 1900's and called it "urushiol"—a name currently used to describe poison ivy's toxic agent as well. Many other trees, shrubs, and plants contain similar noxious compounds.

Skin poisoning of this kind is not only uncomfortable, it can be fatal. As in the case of severe skin burns, if too large a portion of the body is covered with blisters, respiration and excretion of poisonous wastes through the pores is impeded. This, in turn, may lead to a fatal toxemia.

New Type of Insulation.—A new insulation against heat, with potential applications in fields where heat and limited insulation space are factors, is under development at the General Electric Research Laboratory, Dr. C. G. Suits, vice president and director of research, disclosed recently. The new insulation still is in the developmental stage and is not yet ready for marketing.

A glimpse into the possible future of this new "thermal barrier" was provided in the prototype "XR-10" refrigerator-freezer which the G-E major appliance division unveiled recently in the Merchandise Mart in Chicago.

"The new insulation is 10 times as efficient as any now in use," the G-E research director said, "and because of this the 'XR-10,' as a possible future development, employs walls only one-half inch thick."

"This would provide much more refrigerated space and permit placing the refrigerator above the kitchen work counter, rather than in an upright floor cabinet as at present," he said.

No details as to the nature or construction of the insulation were revealed.

Marine Corps Body Armor.—U. S. Marine Corps body armor—called "shrap jackets" by American fighting men— is being produced in quantity at the Westinghouse subsidiary, Plywoods-Plastics Corporation in Hampton, S. C.

In revealing that Westinghouse is making armoring material for the Marine Corps, Eugene R. Perry, Micarta Division manager, said the "shrap jackets" get their life-saving properties from three things—the way they are made, and their two basic ingredients, glass cloth and a synthetic resin. This combination enables the material, known as Doron, actually to stop deadly mortar and grenade fragments and similar low velocity missiles.

When a fragment or a small caliber bullet strikes the material, the layers of impregnated glass cloth separate as the missile attempts to smash its way through. This delamination actually sets up a cushioning effect that instantly reduces the velocity of the fragment to practically zero.

Doron has been in post World War II volume production for only about

two years, but it has already won the confidence and gratitude of the thousands of American soldiers and Marines who wear it for protection in the front lines.

Reports indicate that the armor has been 90 to 95 per cent successful in preventing serious wounds of the torso. However, the Navy Bureau of Medicine and Surgery, which worked with Westinghouse in the development of Doron during the latter part of World War II, said full statistics on the real effectiveness of the armor will not be available until a survey now under way in Korea is completed.

Early versions of the armored vests consisted of flat Doron plates. Some of the flat material was used in utility jackets worn by Marines in mopping up operations in Okinawa during the war in the Pacific. However, tests proved that the flat plates afforded less mobility. As a result, Doron is now curved so that it fits the contour of the body. The plates themselves, $\frac{1}{8}$-in. thick and $5\frac{1}{4}$-in. square, are sewn into the pockets of heavy nylon vests. About 20 plates go into each vest and overlap each other like fish scales.

Doron is made by applying heat and pressure to layers of resin-impregnated glass cloth. While the exact details of manufacture are withheld for security reasons, it can be revealed that the layers are not pressed together tightly. If they were, the plates would lose some of their effectiveness against the missiles they are intended to stop.

The present model of the armored vest, weighing less than eight pounds, has been described as the best of its type ever developed.

New Kind of Optical Wedges.—The principle that makes the earth warm in summer and cold in winter has been employed by a General Electric Research Laboratory scientist to construct a unique apparatus for making optical wedges.

Such wedges, opaque at one end and shading to transparency at the other, control and measure light intensity in optical experiments.

In search of a means for measuring the light intensity and thus the temperature of rocket-motor flames, Dr. Herbert M. Strong, a laboratory physicist, could find no commercially-produced wedges of the proper size and type.

He then hit upon the idea of exposing photographic film on the same principle that makes the sun's rays hot in summer when the sun is high overhead, and cool in winter, when the sun is low in the sky. Dr. Strong constructed a carefully-curved surface to hold the film so that parallel rays of light could strike it at varying angles over its entire length.

The device, recently patented, is equipped with a lamp and lens to supply horizontal rays of light in a parallel pattern. This light strikes the near end of the film directly, but only grazes the far end. The result is a film strip which is completely transparent at one end, but gradually shades into complete opacity at the other.

By changing the shape of the curved film holder, densities in the film can be varied to suit changing requirements. Once set up, the device can produce many such wedges with little effort and at slight cost.

Ordinarily these wedges are made of optical glass by a painstaking and expensive process. Some have been made on film by slowly withdrawing a cover from the film strip, with a light shining vertically on the strip.

Dr. Strong's method, however, is less time consuming and expensive, and is believed to be more accurate as a method of obtaining a desired gradation of exposure.

Optical wedges are used extensively in research for measuring and comparing the brilliance of light sources. They also are valuable tools for calibrating the sensitivity of photographic film.

New Automatic Pipetting System.—A new device, which automatically dispenses controlled volumes of sterile liquids or chemical solutions at predetermined intervals with extreme accuracy, has been announced by Central Scientific Co., of Chicago. Called the Maisch Automatic Pipetting System, it consists of a motorized, stainless steel metering pump, an automatic recycling timer, and a pipette control unit. This completely automatic unit facilitates dispensing of serums, pharmaceuticals, reagents or other industrial solutions that may be required in laboratory or process control. It is ideal for filling ampules with biological fluids. The Maisch pipette unit responds instantly to split-second control of any precision timing switch having adjustable ON and OFF cycle controls. The instant a desired volume of liquid has been dispensed, the pipette control automatically recirculates the flow from the metering pump back into the pump intake, maintaining an average accuracy of 0.5 to 1 per cent for the volumes being dispensed. Any of the various Maisch metering pumps supply a continuously uniform and non-pulsating flow of liquid which is a "must" for accurate dispensing.

The amount of fluid dispensed during each cycle is determined by the speed and capacity of the pump selected, the operating head pressure and time interval to which the pipette control is set. Thus, a pump dispensing at the rate of 20 ml. per sec. under any given head pressure will deliver 100 ml. when the timer is set for 5 sec.

The motorized transmission units for this system are available with fixed speeds for output capacities from 11 to 150 ml. per sec.; also with governor-controlled variable speed universal motors for adjustable output capacities from 3.4 to 30 ml. per sec., and variable speed transmissions for adjustable capacities from 0 to 150 ml. per sec.

Maisch Stainless Steel Pump Heads with oilless bearings feature a positive displacement gear type construction. The newly developed quick demountable heads are interchangeable on all standard Maisch transmissions. They are sterilizable by steam autoclaving for dispensing biologicals and other sterile solutions with a uniform non-pulsating flow. Output capacities from 0 to 150 ml. per sec.

The automatic recycling timer is equipped with adjustable controls for regulating *on* and *off* pump dispensing cycles through the Maisch pipette control unit. This provides a completely automatic time control through practically any required cycle with split-second accuracy. Single cycle timers are also available for control of dispensing cycles only.

The Maisch Pipette Control Unit automatically recirculates the liquid back through the pump the instant a required volume has been dispensed into the receptacle. It is equipped with an electric solenoid and rubber tube pinch valve arrangement which shuts off or releases the flow. It is designed for continuous operation under full load with 110/115-volt, 60-cycle a-c. power.

Voice-Operated Telephone.—An electronic telephone device which can understand and intelligently react to spoken numbers has been built at Bell Telephone Laboratories, research and development organization of the Bell System.

Named "Audrey," a contraction of "automatic digit recognizer," the special circuit automatically determines which of ten numbers, from "1" through "0", has been spoken into an ordinary telephone. In its present form, it responds by flashing an appropriate light, but could equally well control other operations such as dialing mechanisms.

Ultimately, it is hoped to extend Audrey's vocabulary to include additional sounds—words other than numbers. Perhaps Audrey may even be taught to say a few words on command—words maybe, such as "go" and "stop."

The nerve system of the device involves much of the same sort of equipment used in a telephone central office, particularly relays, which are the heart of the modern dial system.

To simplify a very complex electronic process, the newly-constructed recognizer operates on a principle of memory and matching. In other words, it listens to the human voice, then sorts the speech sounds into electrical categories which conform in their own medium to sound-wave patterns. These categories are analyzed by matching against a memory cell containing standard reference patterns already drawn electronically. When the standard pattern is found which best matches the electrical pattern of the spoken number, the appropriate light flashes on.

When the device is adjusted for best performance with a particular speaker, it operates with remarkable accuracy. But it is not yet in a state to answer to a variety of talkers reliably unless it is readjusted for each one. And it will not tolerate careless enunciations or accents.

The automatic recognizer is the subject of a technical paper published in the *Journal of the Acoustical Society of America* by K. H. Davis, R. Biddulph and S. Balashek, members of the Bell Laboratories technical staff who have worked out its intricate details.

In making public various aspects of the unusual type of telephone circuit, the three are careful to point out that the time is still far in the future when a person may be mechanically connected with the party he is calling simply by speaking the desired number into the mouthpiece of his telephone.

Voice-operated devices have been used in the Bell System for many years in transoceanic telephony. They have also proved especially useful in the suppression of electrical "echoes" which sometimes appear in very long telephone connections. Until now, however, the devices have not had to distinguish between different words.

For some time scientists have been able to translate speech into visible form with machines such as the sound spectrograph which was developed at Bell Laboratories. From studies of the resultant patterns, the individual characteristics of the components of speech have been determined.

These characteristics are relatively consistent and fall into groups similar in their outlines when repeated. This is, in substance, the key which enables the recognizer to match essential elements in the spoken sound against a set of ten "standards" determined from studies and then decided on the best match.

This sensitive apparatus is remarkably compact considering the elaborate steps required to simulate the human thought and reaction processes. The experimental equipment fits into a bay roughly the dimensions of a large console TV-radio-phonograph and it could be considerably reduced.